Statistics Canada
www.statcan.gc.ca

SEEA Extensions:

Input-Output Modelling

SEEA Training Seminar for ESCAP

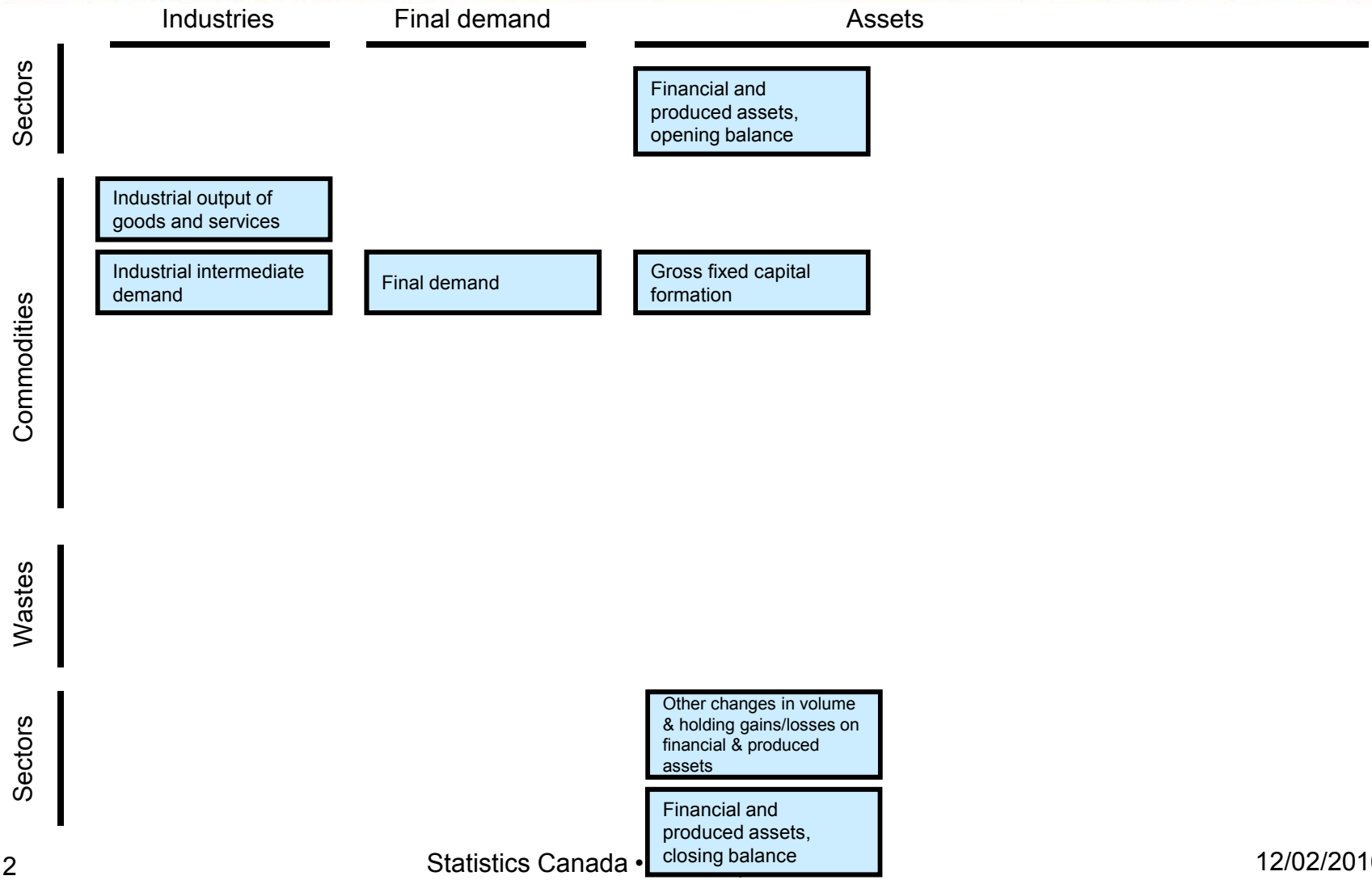
February 23-26, 2016

Chiba, Japan

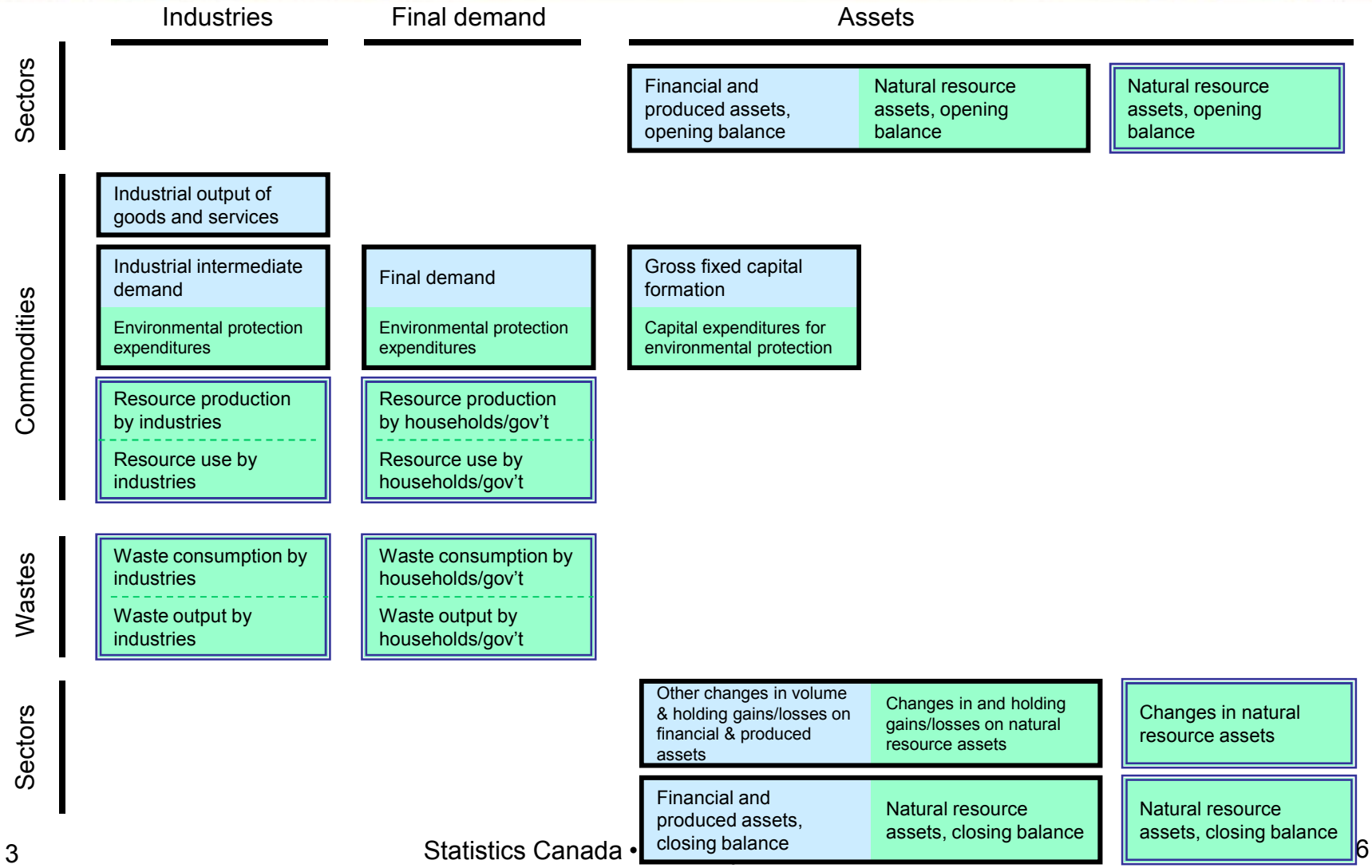
Joe St. Lawrence

Statistics Canada

SNA framework



SEEA framework



Flow Accounting

Table 3.1
General physical supply and use table

Supply table

	Production; generation of residuals		Accumulation			
	Production; generation of residuals by industries (including household production on own account), classified by ISIC	Generation of residuals by households	Industries —classified by ISIC	Flows from the rest of the world	Flows from the environment	Total
Natural inputs					A. Flows from the environment (including natural resource residuals)	Total supply of natural inputs (TSNI)
Products	C. Output (including sale of recycled and reused products)			D. Imports of products		Total supply of products (TSP)
Residuals	I1. Residuals generated by industry (including natural resource residuals)	J. Residuals generated by household final consumption	K1. Residuals from scrapping and demolition of produced assets	L. Residuals received from rest of the world	M. Residuals recovered from the environment	Total supply of residuals (TSR)
	I2. Residuals generated following treatment		K2. Emissions from controlled landfill sites			
Total supply						

United Nations, 2012, *System of Environmental-Economic Accounting: Central Framework*, New York.

Flow Accounting

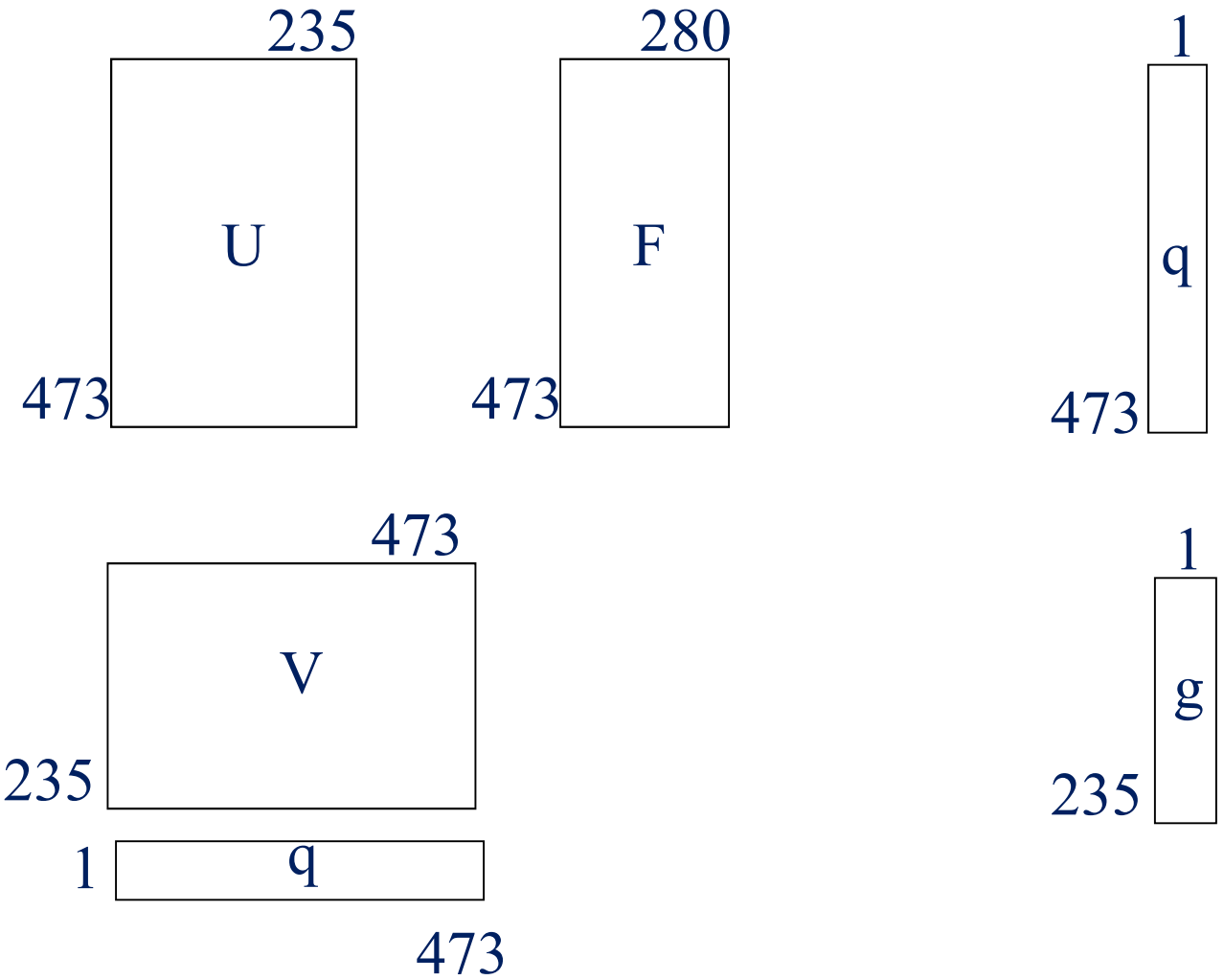
Use table

	Intermediate consumption of products; use of natural inputs; collection of residuals	Final consumption[a]	Accumulation			
	Industries—classified by ISIC	Households	Industries —classified by ISIC	Flows to the rest of the world	Flows to the environment	Total
Natural inputs	B. Extraction of natural inputs B1. Extraction used in production B2. Natural resource residuals					Total use of natural inputs (TUNI)
Products	E. Intermediate consumption (including purchase of recycled and reused products)	F. Household final consumption (including purchase of recycled and reused products)	G. Gross capital formation (including fixed assets and inventories)	H. Exports of products		Total use of products (TUP)
Residuals	N. Collection and treatment of residuals (excluding accumulation in controlled landfill sites)		O. Accumulation of waste in controlled landfill sites	P. Residuals sent to the rest of the word	Q. Residual flows to the environment Q1. Direct from industry and households (including natural resource residuals and landfill emissions) Q2. Following treatment	Total use of residuals (TUR)
Total use						

[a] No entries for government final consumption are recorded in physical terms. All government intermediate consumption, production and generation of residuals is recorded against the relevant industry in the first column of the PSUT.

United Nations, 2012, *System of Environmental-Economic Accounting: Central Framework*, New York.

Monetary Input-Output tables



Numerical example (U: Use)

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
parts	10	5	5	5	10
advertising	5	15	20	40	20

Numerical example (V: Make)

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
parts	10	5	5	5	10
advertising	5	15	20	40	20

MAKE (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	100	0	100	10	0	0	0	0
mines	0	100	0	0	1000	0	0	0
food manuf.	0	0	80	200	0	0	0	0
other manuf.	0	0	0	0	10	150	200	0
services	0	0	0	0	0	0	0	100

Numerical example (F: final demand)

Final Demand
0
0
60
200
600
0
165
0

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
parts	10	5	5	5	10
advertising	5	15	20	40	20

MAKE (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	100	0	100	10	0	0	0	0
mines	0	100	0	0	1000	0	0	0
food manuf.	0	0	80	200	0	0	0	0
other manuf.	0	0	0	0	10	150	200	0
services	0	0	0	0	0	0	0	100

Numerical example
(g: gross industry output / q: gross commodity output)

USE (U)	farms	mines	food manuf.	other manuf.	services	Final Demand
cattle	10	0	80	10	0	0
iron ore	0	0	0	100	0	0
milk	10	5	100	0	5	60
cheese	0	5	0	0	5	200
fuel	100	200	10	50	50	600
steel	0	5	0	145	0	0
parts	10	5	5	5	10	165
advertising	5	15	20	40	20	0

MAKE (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising	g
farms	100	0	100	10	0	0	0	0	210
mines	0	100	0	0	1000	0	0	0	1100
food manuf.	0	0	80	200	0	0	0	0	280
other manuf.	0	0	0	0	10	150	200	0	360
services	0	0	0	0	0	0	0	100	100
q	100	100	180	210	1010	150	200	100	

Σ (across rows)

Σ (down columns)

Numerical example (overview)

USE (U)	farms	mines	food manuf.	other manuf.	services	Final Demand
cattle	10	0	80	10	0	0
iron ore	0	0	0	100	0	0
milk	10	5	100	0	5	60
cheese	0	5	0	0	5	200
fuel	100	200	10	50	50	600
steel	0	5	0	145	0	0
parts	10	5	5	5	10	165
advertising	5	15	20	40	20	0

MAKE (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising	g
farms	100	0	100	10	0	0	0	0	210
mines	0	100	0	0	1000	0	0	0	1100
food manuf.	0	0	80	200	0	0	0	0	280
other manuf.	0	0	0	0	10	150	200	0	360
services	0	0	0	0	0	0	0	100	100
q	100	100	180	210	1010	150	200	100	

The IO model

- ***Basic identity: supply = demand***

$$q + inv_{-} + m = u + fd + x + inv_{+}$$

By substituting for market share ($D=V/q$) and technology ($B=U/g$), we get:

$$g = (I - DB)^{-1}De$$

- ***Allows an estimate of the gross production (g) required from each industry to satisfy a given final demand (e) based on pre-defined relationships of market-share (D) and technology (B)***

Numerical example (B: Technology)

B (U/g)	farms	mines	food manuf.	other manuf.	services
cattle	0.05	0.00	0.29	0.03	0.00
iron ore	0.00	0.00	0.00	0.28	0.00
milk	0.05	0.00	0.36	0.00	0.05
cheese	0.00	0.00	0.00	0.00	0.05
fuel	0.48	0.18	0.04	0.14	0.50
steel	0.00	0.00	0.00	0.40	0.00
parts	0.05	0.00	0.02	0.01	0.10
advertising	0.02	0.01	0.07	0.11	0.20
Σ	0.6	0.2	0.8	0.97	0.9

Numerical example (D: Market share)

B (U/g)	farms	mines	food manuf.	other manuf.	services
cattle	0.05	0.00	0.29	0.03	0.00
iron ore	0.00	0.00	0.00	0.28	0.00
milk	0.05	0.00	0.36	0.00	0.05
cheese	0.00	0.00	0.00	0.00	0.05
fuel	0.48	0.18	0.04	0.14	0.50
steel	0.00	0.00	0.00	0.40	0.00
parts	0.05	0.00	0.02	0.01	0.10
advertising	0.02	0.01	0.07	0.11	0.20
Σ	0.6	0.2	0.8	0.97	0.9

D (V/q)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	1.00	0.00	0.56	0.05	0.00	0.00	0.00	0.00
mines	0.00	1.00	0.00	0.00	0.99	0.00	0.00	0.00
food manuf.	0.00	0.00	0.44	0.95	0.00	0.00	0.00	0.00
other manuf.	0.00	0.00	0.00	0.00	0.01	1.00	1.00	0.00
services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Σ	1	1	1	1	1	1	1	1

Numerical example (The inverse)

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

Meaning: dollars of output from industry at row to deliver (to final demand) a dollar of output from industry at column

Numerical example (De)

Final Demand
0
0
60
200
600
0
165
0

De
42.86
594.06
217.14
170.94
0.00

= 60* 0.56 + 200* 0.05

Actually = 60*0.555 + 200*0.0476

(display table is rounded)

D (V/q)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	1.00	0.00	0.56	0.05	0.00	0.00	0.00	0.00
mines	0.00	1.00	0.00	0.00	0.99	0.00	0.00	0.00
food manuf.	0.00	0.00	0.44	0.95	0.00	0.00	0.00	0.00
other manuf.	0.00	0.00	0.00	0.00	0.01	1.00	1.00	0.00
services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Σ	1	1	1	1	1	1	1	1

Numerical example: $(I-DB)^{-1}De$

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

De
42.86
594.06
217.14
170.94
0.00

Numerical example: $(I-DB)^{-1}De$

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services	De		g=$(I-DB)^{-1}De$
farms	1.11	0.01	0.65	0.08	0.12	42.86	47.5114	210
mines	0.74	1.26	0.50	1.13	1.01	594.06	7.0069	1100
food manuf.	0.04	0.01	1.22	0.03	0.12	217.14	141.185	280
other manuf.	0.13	0.03	0.14	1.80	0.27	170.94	14.297	360
services	0.07	0.03	0.16	0.27	1.32	0.00	0	100
							210	

decompostion of row one of $(I-DB)^{-1}$ times column one of De (Row 1 of $(I-DB)^{-1}$ times column of De)

1.11 * 42.86 ($1.11 of production from farms is required to deliver a dollar of production from farms, so to get 42.86 of final demand we need farms to produce 47.5 dollars of output)

Plus: 0.01 * 594.06 ($0.01 of production from farms is required to deliver a dollar of production from mines, so to get 594.06 of final demand we need farms to produce 7 dollars of output)

Plus: 0.65 * 217.14 ($0.65 of production from farms is required to deliver a dollar of production from food manufacturers, so to get 217.14 of final demand we need farms to produce 141.2 dollars of output)

Plus: 0.08 * 170.94 ($0.08 of production from farms is required to deliver a dollar of production from other manufacturers, so to get 170,94 of final demand we need farms to produce 14.3 dollars of output)

Plus: 0.12 * 0 ($0.12 of production from farms is required to deliver a dollar of production from services, so to get 0.00 to final demand we need farms to produce 0 dollars of output)

So, to deliver all of final demand, farms must produce 210 dollars of gross output

Numerical example

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

De
42.86
594.06
217.14
170.94
0.00

g calc	$g=(I-DB)^{-1}De$
	210
	1100
	280
	360
	100

real g	g
	210
	1100
	280
	360
	100

Calculating output required for different categories of demand…

$$g = (I - DB)^{-1}De$$

- *Recall that this provides an estimate of the gross output (g) required from each industry to satisfy a given final demand (e) based on pre-defined relationships of market-share (D) and technology (B)*

- *So, if I wanted to calculate the gross output from each industry required to produce the demand related to household personal expenditure, I could calculate...*

$$g_{pe} = (I - DB)^{-1}De_{pe}$$

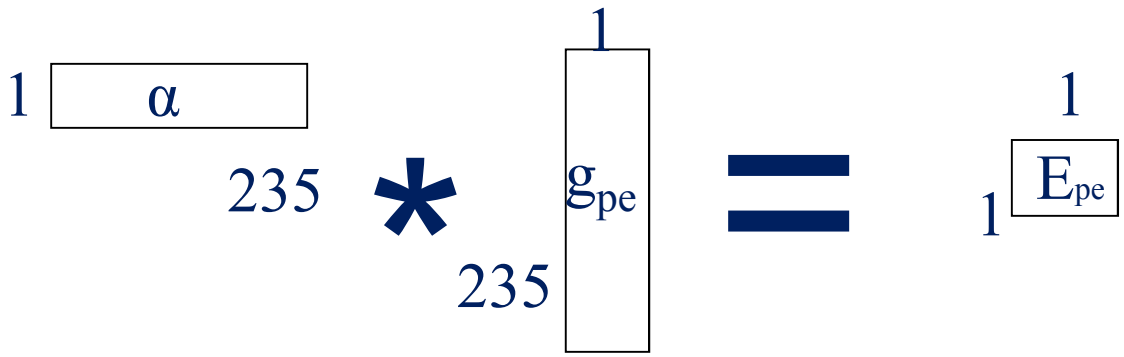
Integration – getting the environment in there…

$$\alpha = E/g$$

$$\underset{235}{\overset{1}{\boxed{\alpha}}} = \underset{235}{\overset{1}{\boxed{E}}} \Big/ \underset{235}{\overset{1}{\boxed{g}}}$$

Sample results – attribution to demand

$$g_{pe} = (I - DB)^{-1} De_{pe}$$



Demand-based perspectives

Table 153-0129[1, 2, 5]

Physical flows by final demand category

annual

Data table | Add/Remove data | Manipulate | Download | Related information | Help

The data below is a part of CANSIM table 153-0129. Use the Add/Remove data tab to customize your table.

Selected items [Add/Remove data]

Geography= Canada
Flow= Greenhouse gas emissions by final demand category (kilotonnes)

Sector	2009	2010	2011
Total, industries and households	714,937	727,805	732,927
Personal expenditure (households)[4]	313,692	314,140	310,336
Non-profit institutions serving households' consumption expenditure	6,153	5,678	5,900
Government net current expenditure	44,184	45,857	45,642
Gross fixed capital formation	74,832	82,694	82,693
International exports	276,077	279,436	288,356

Sample results – direct and indirect intensities (multipliers)

$$1_{\alpha_{235}} * \left(I - DB\right)^{-1}_{235}{}^{235} =$$

Table 153-0115 [1, 2, 3, 4, 5, 6, 8]

Direct plus indirect energy and greenhouse gas emissions intensity, by industry

annual

Data table | Add/Remove data | Manipulate | Download | Related information | Help

The data below is a part of CANSIM table 153-0115. Use the Add/Remove data tab to customize your table.

Selected items [Add/Remove data]

Geography = Canada
Intensity = Direct plus indirect energy intensity (gigajoules per thousand current dollars of production)

Sector	2011
Total, industries	4.91
Crop and animal production [BS11A00]	11.28
Forestry and logging [BS11300]	7.74
Fishing, hunting and trapping [BS11400]	7.49
Support activities for agriculture and forestry [BS11500]	9.71
Oil and gas extraction [BS21100]	13.48
Coal mining [BS21210]	5.71
Metal ore mining [BS21220]	4.32
Non-metallic mineral mining and quarrying [BS21230]	5.76
Support activities for mining and oil and gas extraction [BS21300]	6.32

Numerical example: Multipliers

emissions per unit of output (direct emissions intensity)					
α	farms	mines	food manuf	other manu	services
co2/g	0.5	0.8	0.3	0.4	0.1

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

alpha inverse				
1.218388	1.030774	1.234589	1.698604	1.139405

Numerical example: Multipliers

emissions per unit of output (direct emissions intensity)

α	farms	mines	food manu	other manu	services
co2/g	0.5	0.8	0.3	0.4	0.1

alpha inverse

	1.218388	1.030774	1.234589	1.698604	1.139405

$(I\text{-}DB)^{-1}$

	farms	mines	food manu	other manu	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manu	0.04	0.01	1.22	0.03	0.12
other manu	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

So, $\alpha*(I\text{-}DB)^{-1}$ for row of α times column 1 of $(I\text{-}DB)^{-1}$ is...

	0.5 * 1.11 (1.11 dollars of production from farms is required to deliver a dollar of production from farms.	
	Farms emit .05 CO2 per dollar of output,so in terms of emissions this is:)	0.5543
plus	0.8 * 0.74 (0.74 dollars of production from mines is required to deliver a dollar of production from farms.	
	Mines emit .8 CO2 per dollar of output, so in terms of emissions this is:)	0.59495
plus	0.3 * 0.04 (0.04 dollars of production from food manuf. Is required to deliver a dollar of production from farms.	
	Food manuf. Emit .3 CO2 per dollar of output, so in terms of emissions:)	0.01171
plus	0.4 * 0.13 (0.13 dollars of production from other manuf. Is required to deliver a dollar of output from farms.	
	Other manuf. Emit .4 CO2 per dollar of output, so in terms of emissions:)	0.05075
plus	0.1 * 0.01 (0.01 dollars of production from services is required to deliver a dollar of output from farms.	
	Services emit .1 CO2 per dollar of output, so in terms of emissions this is:)	0.00668
	This is the total emissions required (direct plus indirect) from all industries per dollar of output from farms.	**1.21839**

this basically converts emissions per unit of output (direct intensity) to total emissions required to deliver a unit of final demand (direct plus indirect intensity)

Questions?

Joe St. Lawrence

Environment, Energy and Transportation Statistics
Statistics Canada / Government of Canada
joe.stlawrence@canada.ca / Tel: 613-882-8598

Statistique de l'environnement, l'énergie et des transports
Statistique Canada / Gouvernement du Canada
joe.stlawrence@canada.ca / Tél: 613-882-8598