Minding Ps and Qs of Natural Capital Accounting: Sorting Out Prices and Sustainability Concepts

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Abstract

Change in real wealth is a leading candidate for a headline sustainability measure. The measurement of changes in real wealth raises three questions: (1) what price concept should be used to measure changes in real wealth, (2) how do measurements necessary for change-in-wealth measures relate to standard national economic accounts measurements, and (3) what concept of sustainability does a change in real wealth address? These questions are especially acute in the context of natural capital. Resolving the first question precisely shows that the ideal price concepts used in the national accounting exchange value framework and welfare economics are the same. Prices reflect the marginal contribution to income measured at a point in time and change in prices reflect the changes in substitution or complementarities over time. However, while the two traditions may differ in scope, this is a practical rather than conceptual issue. Second, because prices change over time to reflect changes in substitution and true income effects, simple comparisons of observed prices through time do not provide measures of real changes in wealth or welfare. However, once adjusted, using superlative indices, real or volumetric measures of change in exchange value wealth align with Hicksian welfare measures. This bridges national accounting concepts and welfare-based benefit-cost concepts, opening the door to using standard non-market valuation techniques for marginal units to provide estimates of prices for national accounts purposes, with some satisfiable caveats. These prices can be used within superlative indices. Third, change-in-wealth sustainability measures generally reflect changing patterns of substitution and complementarities, making them intermediate to the classic strong and weak sustainability measures.

<200 words

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1. Introduction

More than 50 years have passed since Nordhaus and Tobin (1972) attempted to measure sustainable consumption and 30 years since the publication of "Our Common Future," which put the concept of sustainable development in the spotlight, (World Commission on Environment and Development, 1987). Concerns about the environment and natural resource depletion were driving factors for these efforts. In recent decades, there has been rapid progress advancing concrete proposals for measuring whether or not the opportunity set being left for future generations is being maintained. Hamilton and Clemens (1999) summarized the economics approach to sustainable development well in that, "however defined, achieving sustainable development necessarily entails creating and maintaining wealth." Multiple leading scholars have shown that non-declining wealth is the foundational measure for non-declining welfare and protecting means for future generations (Arrow et al., 2004; Dasgupta, 2001; Hamilton and Clemens, 1999). Others have emphasized positive changes in net real income (Sefton and Weale, 2006), which would balance with changes in real wealth in a complete accounting system (Weitzman, 2016). Many of these ideas are being incorporated into the proposed 2025 revision of the System of National Accounts (SNA) (United Nations and ISWGNA2024) – the international statistical standard for national economic accounts. The revised standard now explicitly recognizes the role of accounting in measuring wellbeing and sustainability using the national accounts, emphasizing sustainability's link to wellbeing by accounting for changes in resources and capital for future generations.[1] And, environmental and natural resource concerns continue to be a driving feature of this evolution.

A key feature of these inclusive, comprehensive, or genuine wealth measures is that they include a broad set of assets and are not restricted to assets subject to regular market transactions or traditional notions of "fixed capital." Of specific concern is natural capital, which authors focusing on theory have long assumed to be included in measurement systems (e.g., Weitzman, 1976). Indeed, half a century ago, Kuznets (1973) provided an insightful caveat by arguing that measures of "economic growth" (i.e., gross domestic or national product) are not relevant for "countries [that] can provide increasing income to their populations because they happen to possess a [natural] resource" and that it is important to address "the so far hidden but clearly important costs… [of] pollution." Clearly, this anticipates the importance of understanding the costs associated with climate change (Nordhaus, 2019).

[1] Specifically, Chapter 2 of the proposed 2025 SNA (version 7) defines sustainability in the context of the national accounts as follows: "…the measurement of 'sustainability' refers to accounting for the stock and changes in stock of a range of capitals, namely produced, natural, human and social capital. Thus, the measurement of sustainability focuses on the extent to which there is the maintenance and generation of resources to support the economic wellbeing of households in the future" (SNA 2025, §2.17). The chapter refers to natural capital as described in the System of Environmental-Economic Accounting (SEEA), which "facilitates a broader recording of wellbeing in addition to broader measures of natural capital and its sustainability" (SNA 2025, §2.19).

There are a growing number of attempts to operationalize wealth accounting in practice (Dovern et al., 2014; Lange et al., 2018; Managi and Kumar, 2018; OSTP et al., 2023; Smith, 2018), and they all acknowledge the importance, and general undercounting, of natural capital. Imputing prices for natural assets not traded in markets is often held up as a core barrier to implementation of a wealth accounting system (e.g., Smulders, 2012). Contributions by Fenichel and Abbott (2014), Yun et al. (2017), Fenichel et al. (2018), and others have charted a clear path toward measuring accounting prices (or revealed shadow prices).[2] A feature of this price imputation approach is that it allows the price of a stock to change through time as a function of the relative quantities of all assets. Finally, index number theory is an essential part of this approach, providing a foundation for "hope of being able to compare monetary values in a meaningful way when prices and quantities change" (Fleurbaey and Blanchet, 2013).

We focus on natural capital to contribute to the literature on the economics of sustainability by addressing three questions:[3] (1) what prices and approaches should be used to measure *changes* in real wealth, (2) how do measurements necessary for change-in-wealth measures relate to standard national economic accounts measurements, and (3) what notion of sustainability does change in real wealth address?

Answering the first question precisely shows that the balance sheets within the sequence of economic accounts of the SNA, while incomplete, can provide a meaningful starting point for assessing a real change in wealth, despite SNA production boundary seeming to exclude measurement of consumer surplus, which is often the core measurement for valuation from a welfare perspective. This boundary issue creates a potential gap, in *nominal* terms, between national accounts valuation of natural capital assets and welfare analysis often employed in environmental economics.[4] However, by assessing changes in wealth in *real* terms, this gap effectively vanishes, uniting the two approaches under certain satisfiable conditions. Indeed, precisely answering questions (1) and (2) provides a path to bridge existing knowledge of nonmarket valuation in the benefit-cost context with national accounting valuation standards (acknowledging that the change in real welfare is often framed as a "social benefit-cost analysis"). We show that change in wealth and net national income are closely connected to a concept of social or national Hicksian income and can be thought of a special case of national equivalent variation. A key contribution is this paper is that we describe how this connection opens doors to make fuller use of

[2] Fenichel et al. (2018) discuss the various jargon used to describe these prices.
[3] Many of our points are generalizable to produced capital and to income and output.
[4] For example, discussing expenditures on food in nominal terms, the 2008 SNA describes how consumer surplus is omitted from its scope: "…the valuation basis in the SNA is the price paid for the food with no adjustment for the qualitative benefits derived from its consumption. The most that can be claimed for treating expenditure as a measure of welfare is that it may be a reasonable lower bound on the level of welfare engendered by the expenditure" (SNA 2008, §1.77). The System of Environmental Economic Accounting (SEEA) manuals share a similar limitation in scope, stemming from the SNA's valuation principles.

some nonmarket valuation techniques, particularly those that measure the change in consumer surplus, or Hicks compensated measures, associated with a change in a price or quantity.

Finally, answering the first question precisely helps answer the third question about sustainability concepts. It reveals that change-in-wealth sustainability measures generally reflect changing patterns of substitution and complementarities, making them intermediate to the classic strong and weak sustainability measures. Change in real wealth, as a sustainability measure, does not rely on ex-ante assumptions about what is, and is not, substitutable. This puts change-in-real-wealth sustainability measures on a path to address Solow's 1993 plea "for a few numbers, even approximate numbers and away from pronuciamentos" to address sustainable development questions and to move debate away from weak versus strong sustainability arguments.

2. Price and Value

A key purpose of accounting is to track changes in more than one asset, good, or service through time. A core challenge to wealth accounting is to resolve the fact that different physical stocks have different measurement units. Economic theory resolves this by aggregating in the units of value. Since value depends on some aggregation of individual preferences over quantities, marginal values enable a normalization that facilitates aggregation. Therefore, wealth accounting requires measures of marginal value, revealed shadow prices, or accounting prices (as often used interchangeably) for valuing changes in natural capital (Smulders, 2012). Prices are *marginal* values; the question of the right price concept is not separable from the question of the value concept.

There are two relevant value concepts for the purposes of valuing natural capital and thus total wealth. They are "exchange value" and "welfare value" (Obst et al., 2016) – sometimes referred to in terms of rectangles and triangles. Exchange value comes from national accounting (European Commission et al., 2009). Welfare value is associated with welfare economics, and specifically the Kaldor-Hicks criteria common in benefit-cost analysis for environmental goods (e.g., Freeman, 2003). Prior to fully defining these value concepts it is helpful to understand commonalities that exchange and welfare value share to highlight the distinctions between them.

Exchange and welfare value concepts have numerous similarities. Both value concepts use the units of the numeraire. For ease of exposition assume the numeraire is dollars. The numeraire unit can be thought of like a square meter. It can be used to measure any area irrespective of whether it is a triangle, rectangle, or a non-algebraic integral. The two valuation concepts also agree that a price is the marginal value or the value of a small change in quantity. Both traditions recognize the importance of general equilibrium and substitution or complementarity effects. Substitution is important and changing patterns of substitution make assessing changes in real exchange value or changes in real welfare value challenging. Neither the exchange value nor welfare value concept necessarily require efficient markets or first-best allocations. Second-best allocations (Lipsey and Lancaster, 1956), in economies full of distortions, can yield perfectly valid welfare value measurements and exchange value measurements, even when market distortions persist.[5] Finally, as we will show below, it is reasonable to expect that changes in welfare and changes in exchange value can closely approximate each other under certain conditions – conditions that are likely to hold reasonably well in practice.

At the same time, there are important distinctions between the exchange and welfare definitions of value. These differences fall into two camps: conceptual and practical. The important conceptual differences are what types of service flows count and the meaning of zero. The exchange value concept is formulated around a cardinal measure with zero seeming to have well-defined meaning. This is in part because the exchange value concept is heavily grounded in measurement and nominal values. The implication of a meaningful zero is that the demand curve is assumed to intersect the price axis so that there is a choke price, which means that the area under the curve can be calculated without the limit concept.[6] For market goods, exchange value is often observed directly and then decomposed into quantity transacted and average price. The scope of measurement (i.e., what counts) for exchange values is determined through a deliberative process of establishing commonly agreed accounting boundaries, like the System of National Accounts (SNA) production boundary and asset boundary (European Commission et al., 2009). What products and industries are similar enough to be aggregated is established through deliberative classification schemes like the North American Industry Classification System (NAICS), UK Standard Industrial Classification (SIC), or the European Nomenclature of Economic Activities (NACE) system.

The welfare value concept stems from welfare economics and is fundamentally about changes in allocation. Welfare value reflects relative change, with no pre-defined zero. The change in welfare is ideally measured by integrating under the Hicksian demand curve, between price levels, to establish the equivalent variation (or alternative Hicksian measure). However, it is common to integrate under the ordinary Marshallian demand curve to approximate equivalent variation with changes in consumer surplus (Phaneuf and Requate, 2017). Computing either equivalent variation or change in consumer

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[5] Chapter 3 in the SNA (2008) explains that the values we measure need not be under ideal circumstances: "a market price should not necessarily be construed as equivalent to a free market price; that is, a market transaction should not be interpreted as occurring exclusively in a purely competitive market situation. In fact, a market transaction could take place in a monopolistic, monopsonistic, or any other market structure. Indeed, the market may be so narrow that it consists of the sole transaction of its kind between independent parties." SNA 2008, §3.119.

[6] Later, we will show that the zero in exchange value may not actually be well defined. The fact that calculations can be made without a limit concept implies that zero quantity implies zero value. If, on the other hand, the demand curve approaches the price axis only in the limit, then zero quantity can imply infinite value. The latter is problematic for most accounting principles, since there can be a large number of stocks that an economic actor may have zero of.

surplus requires explicit consideration of two quantity levels. However, when only one quantity is observed, some authors (mistakenly) report a "total consumer surplus" using zero quantity as the second measure for the change. This can only be justified under vary narrow conditions and can be ill-defined for essential environmental goods like water or breathable air with no choke price. Because welfare value requires a reallocation, the nature of that reallocation matters. Furthermore in welfare analysis, there is no assumption of a first-best price vector, and welfare analysis is capable of comparing among second-best economies. Finally, what counts for welfare is left to the individuals experiencing the reallocation, i.e., welfare theory endeavors to respect 'consumer sovereignty' over what is included in the utility function.

Assume that the scope of measurement used for exchange values is sufficiently inclusive to approximate what consumers would choose to value and the classification scheme provides sufficient refinement. We maintain this assumption through the remainder of the paper because defining what counts is simply a decision that need to be made to implement a useful if imperfect statistical system.[7] Under this assumption, the exchange price (i.e. the marginal exchange value) can be equivalent to the welfare price (i.e. the marginal welfare value). Indeed, the locus of quantity-price pairs making up the welfare theoretic inverse demand curve, could also provide an exchange value inverse demand curve. In the case of a single good, the exchange value at time zero is the rectangle made up of area $a + c + f$ and $f + g$ at time one (Fig 1).

The welfare value cannot be defined without providing a counterfactual quantity, which is assumed not to shift the inverse demand curve (allowing only a shift in quantity demanded and not a shift in demand). If the counterfactual quantity were zero, then the so-called "welfare value" at time zero would be $a + c + f + h$ and at time one would be $a + b + c + d + f + g + h$ (Fig 1). Assuming that the demand curve slopes down, "welfare value" at any point in time always exceeds the exchange value in nominal terms. This is the source of the "rectangles vs triangles" debate, where exchange values are rectangles and welfare values include triangles.

When we instead assess changes in value in real terms, the debate about exchange value and welfare value is not about the price concept and hence not about the value concept or methods of measurement. As we discuss in greater depth below, the debate transforms into being about what counts within measurements of value, because which services are included determines the location and shape of the inverse demand curve. In turn, what gets counted in accounting systems hinges on whether and how different assets, goods, or services are reflected in the classification and taxonomy systems. Economists generally have a good idea how to do most measurements once it is determined what should be measured.

[7] How accounting boundaries are established and the development of classification systems and taxonomies are important, but beyond the scope of this paper.

3. Sustainability and Wealth

Sustainable development requires accounting for the allocation of opportunities through time and perhaps the level of consumption at a point in time to assess whether growth is in fact sustainable. The economics literature primarily focuses on sustainable development requiring non-declining opportunities (per capita) through time. This is usually framed in the modern understanding of consumption, which is more aligned with receiving a flow of services than using up a durable good (Banzhaf, 2023). This leads to the idea that non-declining wealth is a necessary, but not sufficient, condition for sustainable development (Dasgupta, 2001) reflecting concerns that the level or distribution of consumption at a point in time is not directly considered.[8] In what follows, we focus on the allocation of opportunity through time. The opportunity set for a population at a time t, over a vector of capital stocks $\boldsymbol{s}$ is valued as[9]

(1) $$V(\boldsymbol{s}(t)) = \int_t^\infty k(\tau,t) W\left(\boldsymbol{s}(\tau), \boldsymbol{x}(\boldsymbol{s}(\tau))\right) d\tau$$

Where V is the intertemporal welfare function or "value" function, $k(\tau, t)$ is the discounting function, which is effectively an intertemporal equity weight between time periods in the Bergstrom-Samuelson social welfare function (Mueller, 2003).[10] The discounting function is most commonly specified as $k(\tau, t) = e^{-\delta(\tau-t)}$. We follow this convention in what follows.[11] W is the one period net value of services flows function, and $\boldsymbol{x}(\boldsymbol{s})$ is a vector of functions describing the economic program or resource allocation mechanism, described as a feedback rule, that is in place at the evaluated time, τ. At a point in time, the economic program makes use of the vector of stocks to generate production and a flow of services that can be invested (saved) or used for direct consumption and, depending on the accounting boundaries, non-market utility. Eq (1) does not include the maximization operator because the economic program, $\boldsymbol{x}(\boldsymbol{s})$, is pre-specified as a feedback rule on the set of stocks that is found through empirical measurement, and an optimizing economic program is one possible special case. This allows for the fact that that institutions may not maximize social welfare (Dasgupta, 2001; European Commission et al., 2009). Finally, stock s_i in vector $\boldsymbol{s}$ evolves as $\dot{s}_i = G_i(\boldsymbol{s}, \boldsymbol{x}(\boldsymbol{s}))$.This expression is autonomous in time, though time itself can be treated as a stock (Arrow et al., 2003).

3.1. Income and Wealth

[8] The notion is that some normative criteria may require a level of consumption at a point in time.

[9] Distribution is an important issue. Arrow et al. (2003) suggest per capita measures, but there is not a clear ethical reason for per capita weighting within a population (Jorgenson, 2018). Furthermore, when all members share the same quantity of public good dependent on a stock of natural capital, the value may simply scale with population. While these issues are important, they are less so for the contribution of this paper. Therefore, we abstract from these intratemporal distributional issues.

[10] Intragenerational weights have already been dealt with within W.

[11] Equation (1) assumes a time separable structure. The theory appears extendable to recursive structures (Kakeu, 2023), but we have left this for future investigation.

The interpretation of V is fully dependent on the definition of W, which may be interpreted as the "single period income function." However, the concept of income is complicated (Hill and Hill, 2003; Sefton and Weale, 2006; Weitzman, 2016), and it is worth a short diversion to discuss the concept of income. Fisher (1906) defined "real income" as "services" and included "the supplementary elements which we found lacking under the head of money-income… for it [real income] recognizes that money is only an intermediary, and seeks to discover the real elements for which that money-income stands." Fisher's definition emphasizes two key features. First, *services*, which means that intermediate commodities used up in generating services are not counted – i.e. costs to produce the services are not income. Second, income does not have to be in monetary form. Krutilla (1967) emphasizes this latter point in the context of the environment and argues that, "continued availability [of 'scenic wonder or a unique and fragile ecosystem'] are a significant part of the real income of many individuals." These are welfare-increasing services too, and modern economics views the enjoyment of services as consumption even if goods are not "used up" (Banzhaf, 2023). Fisher's definition preserves the need for the concept of assets, and views income as an "interest like return" on wealth (Weitzman, 2016).[12] Fisher's definition of income aligns with common place understanding of the term income.

A slightly different view of income is due to Hicks (1939), whose income concept is defined as that amount that can be consumed at a point in time without impeding future consumption or the maximum amount of consumption that holds capital intact (Heal, 1998; Hill and Hill, 2003). A key challenge, for our purpose, is that Hicks seeks to remove the need to track assets altogether (Weitzman, 2016). In other words, Hick's definition of income goes beyond stock-flow accounting and requires a normalization of the state of consumption to welfare.

The Fisher and Hicks notions of income align under what Weitzman (2016) calls, "perfectly-complete accounting," – a case with no double counting, no undercounting, and elimination of costs, including defensive expenditures – the case where all stocks and services are included whether exchanged in the market or not. Under this complete accounting, wealth is equivalent to the expenditure function associated with a level of welfare and a vector of realized shadow prices, i.e.,

$$\mathcal{W}(t) = \mathcal{W}\Big(\boldsymbol{p}\big(\boldsymbol{s}(t), \boldsymbol{x}(\boldsymbol{s}(t))\big), V(t)\Big) = \mathcal{W}\Big(\boldsymbol{\nabla_s V}\big(\boldsymbol{s}(t), \boldsymbol{x}(\boldsymbol{s}(t))\big), V(t)\Big).$$

So, wealth is a numeraire or money-metric normalization of welfare, just like the expenditure function is a normalization of utility.

There are three important features of this formulation of wealth as the expenditure function. First, the prices are realized shadow prices or accounting prices that map the current set of stocks and the

[12] Hulten (2006) points out that what is capital and what is an intermediate good depends on the accounting period. However, for natural and ecosystem capital, it is clear some asset must persist long enough that calling them intermediate goods seems unreasonable.

economic program. The economic program does not have to be optimal and may provide for a second-best economy.[13] Second, the wealth normalization is a linear extrapolation summing all assets at their current prices, including numeraire savings, which is $\mathcal{W}_0 = \boldsymbol{p}(\boldsymbol{s_0})'\boldsymbol{s_0}$ (Fig 2).[14] Figure 2 shows the case of two stocks: stock, s_i and a numeraire stock, which can be thought of as cash equivalent savings if all other assets could be converted to cash at current prices. Here, the extrapolation implies that if all of stock s_i were converted to cash at the current price, then total cash savings could be $\mathcal{W}_0$. However, it is clear from Figure 2 that any attempt to liquidate stock s_i would lead to price changes in even a somewhat functioning market. Furthermore, the economic program does not imply liquidating stock s_i, making $\mathcal{W}_0$ hypothetical. An alternative interpretation of $\mathcal{W}_0$ is that it is the maximum amount that an investor, sharing society's discounting process, would be willing to pay for the current set of stocks and with the obligation to maintain the current economic program, so that $\mathcal{W}_0 = \int_0^\infty k(\tau,0)W\left(\boldsymbol{s}(\tau),\boldsymbol{x}\big(\boldsymbol{s}(\tau)\big)\right)d\tau$.[15]

However, $\mathcal{W}_0$ should not be interpreted as the welfare level associated with the specific indifference curve, e.g., $V\big(\boldsymbol{s}(0)\big)$. Rather, $\mathcal{W}_0$ is a normalization. It maps the current economic program and quantities of stocks into units of the numeraire as if current conditions can be maintained. For example, an alternative set of prices and quantities on a different welfare indifference curve, e.g., $V\big(\boldsymbol{s}(1)\big)$, could lead to the same level of wealth, but would require a different economic program (assuming all other stocks were as they were at time zero) because points A and B in Figure 2 are not equivalent. This is because the actual level of wealth, like the level of welfare, does not have an absolute zero. This latter point would hold even if one where to only focus on produced capital.

Measures of welfare and wealth are connected to an economic program. Furthermore, if the economic program is anticipated and priced into all assets, then how can wealth change? Wealth can grow by reducing the relative scarcity of preferred goods relative to less preferred goods. Generally, this happens through unanticipated innovation, which generates dividends from human capital, unanticipated resource discovery, or unanticipated net growth of renewable resources, which includes unanticipated or unpriced exhaustion or unanticipated institutional changes. Holding institutions constant, without unanticipated innovation, discovery, or resource net growth, increased scarcity of the preferred good would lead to a rise in price and create incentives for greater supply or creation. Furthermore, the growth would be priced into existing assets with perfect foresight.

Market structures often fail to provide incentives for efficient innovation, discovery, or management of endogenous resource net growth when the flow of services associated with stocks are not

[13] It seems likely, however, that the common integrability conditions (Freeman) still need to be satisfied.
[14] Where $'$ is the horizontal transpose of the vector.
[15] It is in this sense that zero is not actually well defined in the exchange value concept.

fully excludable, leading to a suboptimal response to scarcity signals without changes in complex human institutions (Gordon, 1954; Libecap, 1994; Romer, 1990). This leads to second-best economies, where the measure of changes in wealth matters. Likewise, on the consumer side, declining relative prices of preferred goods leads to income and substitution effects. However, only the income effect contributes to changes in wealth, in the Hicksian sense, because this enables greater substitution in the future without compromising consumption even further into the future. Nevertheless, substitution opportunities, especially from natural resources, often create unaccounted for forms of income. For example, when a wetland protects a house from flooding, resources for flood prevention become available for other uses – providing an income effect that is seldom fully priced in the market (Taylor and Druckenmiller, 2022).

3.2 The relationship between change in welfare and the change in wealth

Non-declining welfare implies $dV/dt \geq 0$. Using the left-hand side of Eq (1) and applying the chain rule, $dV/dt = (\nabla_s V)'\dot{\boldsymbol{s}}$, which states that the change in the intertemporal welfare function is the gradient of the value function with respect to stocks multiplied by the change in stocks. The price of an asset is change in the net present value, (Eq 1), with respect to a change in the stock of the asset (Hulten, 2006; Jorgenson, 1963), so that $\partial V/\partial s^i = p^i$. Therefore,

(2) $$\frac{dV}{dt} = \boldsymbol{p}'\dot{\boldsymbol{s}}$$

Equation (2) says that a change in intertemporal welfare is equivalent to a change in quantity of capital stocks evaluated at the appropriate price vector. Notice that the price vector is held constant.

Using the definition of wealth as equivalent to the budget constraint, wealth is defined as

(3) $$\mathcal{W} = \boldsymbol{p}'\boldsymbol{s}$$

where at least one element of $\boldsymbol{s}$ represents numeraire savings for future consumption. The change in real wealth over a small change in time is the change in the expenditure function assuming constant relative prices, which is

(4) $$\frac{d\mathcal{W}}{dt} = \boldsymbol{p}'\dot{\boldsymbol{s}}$$

Comparing Eq (1) and Eq (3) show that level of wealth must be a normalization of the welfare measure, as shown in the prior section. Further, the level of wealth aligns with the exchange value by summing quantities at specific prices. Again, $\mathcal{W}_0$ is a measure of the possible opportunities available if all stocks could be liquidated without any price effects. This is untenable for an aggregate measure of national wealth if at least one non-cash stock is an essential stock, e.g. water. If wealth is an assessment of future opportunity, then as wealth goes to zero from the right, the nation ceases to exist in any practical sense.

So zero wealth, as such, is undefined as an aggregate concept. Likewise, V is undefined when $s = 0$, because the definition of an essential stock is $\lim_{s \to 0} p \to \infty$.[16]

Comparing Eq (2) and Eq (4) shows that for a sufficiently small changes in t, $\Delta t \to dt$, the change in welfare equals the change in wealth, or $\frac{dV}{dt} = \frac{dW}{dt}$ (Dasgupta, 2001). This provides a pathway to measure changes in sustainability associated with changes in the vector of stocks at constant relative prices. These measurements remain a reasonably good approximation as the change in time gets larger.

In order to measure changes in prices that are consistent with the likely second-best changes in stocks, the price concept cannot impose a first-best optimization assumption; rather, it must depend on the information about the economic program or resource allocation mechanism (Fenichel and Abbott, 2014; Fenichel et al., 2018). The observed economic program contains information about revealed preferences, institutions, transaction costs, as well as the set of market distortions. The measured economic program takes the economic program as the result of a political-economic equilibrium under the prevailing (or most likely) set of institutions and transaction costs. Fenichel et al. (2018) show that the accounting price of stock $p(s)$ (using superscripts as indexes) is

$$(5) \qquad p^i(\boldsymbol{s}) = \frac{\frac{\partial W(s,x(s))}{\partial s^i} + \left(\frac{\partial p^i}{\partial s^i}\dot{s}^i + \sum_{j \neq i} \frac{\partial p^j}{\partial s^i}\dot{s}^j\right) + \sum_{j \neq i} p^j \frac{\partial \dot{s}^j}{\partial s^i}}{\delta - \frac{\partial G^i(s,x(s))}{\partial s^i}}$$

and, they develop a set of numerical approximation methods. If there were only one single stock, then Eq (5) simplifies to $p(s) = \frac{W_s(s) + p_s \dot{s}}{\delta - G_s(s,x(s))}$, where subscripts are partial derivatives. This is an algebraic rearrangement of Jorgenson's (1963) formula for the price of invested capital with taxes set to zero. However, the marginal dividends term, W_s is more general because Jorgenson assumes marginal dividends are constant. In Eq (5) marginal dividends can vary with $\boldsymbol{s}$. The other numerator terms are capital gains (or loss) terms, and by the chain rule $\frac{dp}{dt} = \frac{dp}{ds}\frac{ds}{dt}$.[17] In the denominator δ is the general opportunity cost of capital or discount rate, while the $-G_s$ term is a rate of depreciation that depends on the economic program and dynamics of the stock. Equation (5) generalizes the single stock. The structure of capital gains is important. The middle term represents own and cross price effects. The final numerator term is a cross stock effect. Prices capture information about substitution and complementarity (Yun et al., 2017). These show how unanticipated (unpriced) innovation, discovery, exhaustion, or policy change related to one stock provides the source of potential price changes for other stocks, along with price changes for the stock in question.

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[16] The fact that $V(0)$ is undefined does not prevent measurement of the changes in wealth for $s > 0$. This also means that $\mathcal{W} = 0$ cannot be equal to $V(0)$, calling into question the meaning of $\mathcal{W} = 0$.

[17] Price only depends on time through stock, so the partial derivative is the total derivative.

3.3 Measuring real changes in wealth or welfare for a single asset

Following from Eqs (2) and (4), $\frac{\Delta\mathcal{W}}{\Delta t} \approx \frac{\Delta V}{\Delta t}$ and any error in this approximation vanishes as $\Delta t \to 0$. Assuming that time only affects the value of V through changes in stocks, then following the standard rules of integration, $V(\boldsymbol{s_1}) - V(\boldsymbol{s}_0)$ is the change in V over the time interval [0,1]. It must also correspond to the change in $\mathcal{W}$ over the same time interval.

Measuring the change is V requires knowledge of $V(\boldsymbol{s}_1)$ and $V(\boldsymbol{s}_0)$, which requires the ability to identify the complete vector $\boldsymbol{s}$**,** the interconnections among stocks and their dynamics, the economic program or resource allocation mechanism, and the value of service flows derived from capital. These conditions probably cannot be perfectly satisfied. Still, substantial progress can be made with only information on prices and quantities, which is generally the information most directly available to national statistical offices compiling economic statistics. Equation (5), however, shows that prices will change as stocks change (over time) because of direct effects on the stock or substitution (or complementarity) effects, and substitution effects should not be included in measures of real income, change in wealth, or change in welfare assessments.

While it is possible to measure or impute prices at a point in time using equation (5), it is important to measure the real change in wealth or the change in welfare over time using a vector at constant relative prices to isolate the income effect. A convex combination of prices at time zero and time one is a way of finding an appropriate constant relative price.

(6) $\quad \Delta V = \Delta\mathcal{W} = \sum_i \bar{p}_i \left(s_{i,1} - s_{i,0}\right),$

Where $\bar{p}_i$ is a convex combination of $p_{i,1}$ and $p_{i,0}$.

The end members of a convex combination are $\boldsymbol{p}_1'(\boldsymbol{s}_1 - \boldsymbol{s}_0)$ and $\boldsymbol{p}_0'(\boldsymbol{s}_1 - \boldsymbol{s}_0)$ and correspond to changes computed with Paasche and Laspeyres indexes.[18] When there is more than one stock of interest the Paasche and Laspeyres indexes provide a more robust foundation than working from individual price averages. The Paasche and Laspeyres indexes help keep track of the numeraire stock.[19] Consider the simple arithmetic mean of prices (Fig 1). Taking the arithmetic mean of prices (or of the Passche and Laspeyres indices) is the equivalent to setting:

(7) $\quad \bar{p} = \frac{1}{2}(p_1 + p_0).$

If the demand curve were linear (or could be reasonably approximated with a first-order Taylor approximation), then the two errors induced by using the arithmetic mean, the omission of region b and

[18] Here, we are showing level changes, which is helpful for building intuition. The Paasche and Laspeyres indexes are usually presented as percent changes multiplied by a base level. The formulation here is the same for the case of a single stock plus a numeraire stock.

[19] Dynan and Sheiner (2018) describe a similar example in Box 1 of their paper, illustrating the link between the change in consumer surplus and the change in real GDP.

inclusion of region e perfectly offset each other to yield a change in value equal to $b + d + g$ in Figure (1). Harberger (1971) suggested that linear approximation and used it to derive a second-order approximation to equivalent variation when there are no substitution effects. Hotelling and Hicks also suggested this approximation (Diewert, 1992).[20] Alternatively, it is possible to derive the same calculation by averaging the resulting Paasche and Laspeyres index values, where the Paasche index, which applies period one's price to both periods so that the change in real value is, $(p_1 s_1 - p_1 s_0)$ or $p_1(s_1 - s_0)$ or g, and the Laspeyres index, which applies period zero's prices to both periods, $(p_0 s_1 - p_0 s_0)$ or $p_0(s_1 - s_0)$ or $g + b + d + e + i$. Averaging, we get $\frac{(b+d+e+i+g+g)}{2}$. Since $d = i$ and $b = e$, then $\frac{(2b+2d+2g)}{2}$ or simply $b + d + g$ as the change in real wealth.

The graphic intuition (Fig 1) for the mean price approach relies on there being only a change in quantity demanded, as opposed to a change in demand (so that the ordinary Marshallian demand curve is the Hicks compensated demand curve). Furthermore, measuring the actual change in wealth requires adjusting for the associated changes in holdings of numeraire cash savings, which results in the change in consumer surplus, regions a+b+c+d in Figure 1. Region g represents intermediate rents and not final goods for consumption. Assuming nominal income is the same from period zero to one, then region $a + c$ represents a pure shift from intermediate goods to final consumption. It is common to use the ordinary Marshallian demand function and change in consumer surplus to approximate equivalent variation and Hicksian income (Phaneuf and Requate, 2017; Willig, 1976).

The case of only a shift in quantity demanded is not tenable, and the link to the change in consumer surplus is not intuitive in the national accounting context. What is desired is a measure of change in wealth that is clearly the change in the numeraire or budget constraint at constant relative prices, which aligns to a change in exchange value. If the national accounts were to measure a change in the nominal value of this asset or good, it is clear that it would compare the following "rectangles" in Figure 1: $p_1 s_1 - p_0 s_0 = (f + g) - (a + c + f)$. Yet, given that prices have changed from period zero to one, this is not a meaningful change in real exchange value. Rather, the constant relative price or volumetric change makes an 'apples to apples' comparison across periods by using a common price index. Hence, $\bar{p}(s_1 - s_0) = d + e + g$ reflects the real change in wealth as measured by the averaged Paasche-Laspeyres value. The key result in Figure 1 is that, given $e = b$, the real change in wealth is $b + d + g$ is also equal to the change in welfare. Thus, under these conditions – specifically holding constant relative prices, valuing changes in real wealth in the national accounts aligns with the valuation of a change in welfare associated with the change in quantity $(s_1 - s_0)$. This alignment occurs despite the fact

[20] In the context of wealth accounting, Arrow et al. (2004), and Yun et al. (2017) use this weighting system, with Yun et al. comparing it to exact integrated measures and showing that it yields reasonably accurate results. However, Yun et al. only consider a subset of the possible assets that would go into a complete wealth account.

national accounts focus on measuring "rectangles" of prices times quantities, contrasting with common approaches in environmental-economics focusing on broader measures of "triangles" in valuing changes to welfare.

Finally, considering the hypothetical case where $p_1 = 0$ and rectangles f and g vanish (Fig 1), is helpful for addressing a few additional conceptual issues in the context of national accounts. First, natural capital assets with a current market value of zero (or near zero) can contribute to a positive change in real wealth, despite nominal wealth falling in this scenario. Second, the counterfactual or baseline matters for the relevant valuation in the case of the zero-price asset. If the market value of the counterfactual in time zero was also near zero, then the change in welfare and real wealth are also near zero, even if the *level* of welfare generated is potentially enormous. Finally, in either scenario, when evaluating changes to real wealth or welfare, the triangle h does not factor into our analysis. The takeaway is that, for the purposes of national economic accounting, we do not need to try to value triangle h. Yet, the triangle associated with *marginal* units, $b + d = i + e$, is the relevant triangle that unites welfare economics and national accounts.

3.4 Changes in real wealth and welfare with multiple assets or goods

In the single good or single asset case, a change in real wealth aligns with the change in welfare, but how should these be added up when there is more than one good or asset? Consider the change in wealth between time zero and time one. The change in wealth must be the change in the expenditure function associated with all shifts in quantities and prices from period zero to period one at constant relative prices. This is the equivalent variation (EV, in Figure 2) - how much more numeraire there could be if all stocks were converted to the numeraire, at current prices. This is the distance between $\mathcal{W}_1$ and $\mathcal{W}_0$ along the y-axis in the upper panel of Figure 2.

Diewert (1992) showed that the arithmetic mean of the Paasche and Laspeyres indices is a first-order approximation of equivalent variation or as a second-order approximation if substitution patterns do not change between period zero and one. Diewert (1992) provides an extensive investigation of index number theory and connection between constant real price (or volumetric) measures and equivalent variation. He defines the concept of a superlative index, which is an index that provides an exact second-order approximation to equivalent variation, even if elasticities of substitution differ between period zero and period one. He identifies the Fisher Ideal index, which is the *geometric* mean of the Laspeyres and Paasche indices, as a superlative index. In the single stock case the Fisher Ideal index simplifies to the geometric mean of prices (or quantities). Dumagan (2002) shows that the Tornqvist index is also a superlative index, nearly identical to the Fisher ideal index, and suggests it is easier to use in practice.[21] Tornqvist and Fisher ideal indices are regularly recommended for use in national accounts to compute real changes in gross domestic product or for computing consumer price indices.[22,23]

Consider the case with two stocks (Fig 2), a stock of interest, s_i, and a numeraire stock, n, that represent cash savings, or the cash equivalent, at current prices, of all other savings aside from stock s_i. In time period zero, stock s_i and the cash savings are held at point A, and the implied price of stock s_i at point A is measured. From the measurements, we infer some welfare indifference curve, $V(\boldsymbol{s}(0))$, (at least locally) and the associated budget constraint line or potential expenditure line, which is the line segment $[\mathcal{W}_0, \mathcal{W}_0/p_0(\boldsymbol{s}_0)]$. Assume that unanticipated innovation, discovery, or policy changes for some other good leads the implied price of stock s_i to fall. This could be associated with society acquiring more of stock s_i, perhaps through conservation actions. This change rotates the wealth constraint and leads to the allocation associated with point B.[24] Innovation, discovery, or policy change directed at stock s could have a similar effect. Likewise, the shift to holding more stock s_i implies that all else equal the implied price of stock s_i has fallen. The welfare indifference curve associated with allocation A, must shift outward such that it never crosses the welfare curve associated with level A. Now, if one imposes constant relative prices from period zero on period one, there is a new tangent at point C illustrated by the dashed line. Point C is a hypothetical point. Society is indifferent between points B and C, and point C is implied by the prices from period zero. That is, with an abuse of notation, $V(s_{i,1}|B) - V(s_{i,0}|A) = V(s_{i,1}|C) - V(s_{i,0}|A) = \mathcal{W}_1 - \mathcal{W}_0$, where the stocks and their prices are conditional on the allocation A, B, and C. The difference between $\mathcal{W}_1 - \mathcal{W}_0$, the equivalent variation, represents the wealth gained between period zero and one, measured in real terms to remove substitution effects. This means that $p^*_{s_i} = \frac{\mathcal{W}_1 - \mathcal{W}_0}{s_{i,1} - s_{i,0}}$, which implies $p^*_{s_i}(s_{i,1} - s_{i,0}) = EV = \mathcal{W}_1 - \mathcal{W}_0$. It is the $p^*_{s_i}$ that is the price that would be used in Eq (2) or Eq (4) for the element of p associated with stock s_i.

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21 In the Fisher Ideal and Tornqvist indices price and quantity play the same role mathematically, so it is easy to move between price and quantity indices and between the Fisher Ideal and Tornqvist index (Dumagan, 2002). The distinction is that the Tornqvist index has a calculation of the form $\ln\left(\frac{s_{i,t}}{s_{i,t-1}}\right)$ whereas, the Fisher Ideal calculation reduces to $\frac{s_{i,t} - s_{i,t-1}}{s_{i,t-1}}$ (Dumagan, 2002).

22 See European Commission et al. (2009) or https://www.imf.org/external/np/sta/tegeipi/ch16.pdf. The Fisher and Tornqvist indices are used to compute chained GDP values.

23 For a review of quantity indexes and their role in measuring real GDP and the national accounts more generally, see Dynan and Sheiner (2018) for discussion.

24 The analysis in the section can also be done with relatively fewer holdings of stock s_i or higher implied prices of stock s_i. If prices are also changing for other stocks, then the expenditure function going through point B may not intersect point $\mathcal{W}_0$; the resulting gap represents inflation (Dynan and Sheiner, 2018).

The equivalent variation is well approximated by using the prices and quantities from allocation A at time zero and B as time one in the Fisher ideal index (Diewert, 1992) or Tornqvist index. Plotting the allocations A, B, C for stock s with their prices (Figure 2 panel B) and connecting points A and B provides a linear approximation to the Marshallian ordinary demand curve. Connecting points C and B provides a linear approximation to the Hicks compensated demand curve. The area $a + b$ is the change in consumer surplus. Area $a + b + c$ is the equivalent variation. Area d are rents to producers and generally represent intermediate products that would be subtracted off in a complete accounting of income. Area a is a shift from intermediate goods to final wealth of consumers.

4. Implications of Theory for Wealth Accounting

Sections 2 and 3 connect the national accounting exchange value concept, welfare economics, and index number theory. The ability to make the connection depends on measuring revealed shadow prices based on the actual economic program or resource allocation mechanism. In this section we discuss the implications of the deep connections among national accounting and welfare theory for measuring change in real wealth and interpretation for sustainability.

4.1 Statistical Standards

For the purposes of measuring the change in real wealth there are two main components. The first component concerns measures of vectors of prices and quantities at successive points in time. These data are used to measure the relative prices that underpin the derivation of real wealth in combination with standard national accounting tools, such as the Fisher ideal or Tornqvist indices. In other words, national statistical offices do not need to measure the hypothetical "h triangle" in Figure 1 that accounts for consumer surplus for the initial quantity units around zero. Rather, for the hypothetical "first unit" we only measure its exchange value as if it were the marginal unit, avoiding the need to calculate the (potentially infinite) consumer surplus for the initial unit for assets like water. Adding a third time period can be done by using chaining approaches that are used for other real national income accounting measures.

The second measurement component involves establishing a reference level for wealth. In line with the theory outlined earlier, this requires establishing a hypothetical value for baseline wealth, $\mathcal{W}_0$. Although statistical offices avoid engaging in the measurement of hypotheticals, particularly when such hypotheticals are difficult to objectively measure, in this case the hypothetical aligns perfectly with the idea of the sum of exchange values for assets at a given point in time, where exchange values are price times quantity. This measure of total wealth, while hypothetical in the sense that all assets cannot be sold at once, serves only as a normalization. The hypothetical price-quantity pair at point C in Figure 2 is not used.

The remaining challenges for national accounts are how to measure prices and quantities. These are technical questions, which are addressed later in this section and the determination of accounting boundary and classification questions. Accounting boundary and classification questions hinge on what to count rather than conceptual questions of how to count. Paraphrasing Nordhaus and Tobin (1972), leaving out assets that are changing will give the wrong signals of economic growth and the wrong measures of sustainable development. Insofar as methodological misalignment is an impediment for including missing assets, research connecting welfare economics and national accounts concepts, therefore, has the potential to add to the toolkits of national statistical offices as they produce more meaningful real asset statistics inclusive of natural capital assets. We return to this point in sections 4.3 and 4.4 below.

4.2 Change in wealth and weak versus strong sustainability

Debates related to weak and strong sustainability go back decades. One reason to measure real income or change in real wealth is that real values acknowledge the relationships among stocks that agents in the economy face, which factor in all the substitution and complementary relationships while isolating the real (Hicksian) income effect.

It is not correct to frame change-in-wealth based approaches to sustainability as strictly weak sustainability metrics. Strictly weak sustainability implies unrestricted substitution possibilities and that it is always possible to compensate for the loss of any specific asset with sufficient quantities of another asset. Pearce and Atkinson (1993) associated savings rule measures of sustainability, a precursor to change-in-wealth approaches, with weak sustainability, and the association has stuck. Pearce and Atkinson formulated their analysis in terms of nominal changes in wealth where prices are disconnected from asset quantity. They argue that saving rule approaches allow "unconstrained elasticities of substitution" between natural and produced capital. They suggest this follows from the linear in prices and linear in quantities nature of saving rule approaches – implying enough of asset b can always compensate for the loss of asset a. Furthermore, there is a long history of imposing substitution relationships on natural assets. Nordhaus and Tobin (1972) warn against this. The elasticity of substitution is implicit in measurement of the change in prices, and at times is imposed by the analyst through functional form assumptions used in estimation, and these assumptions are not innocuous Cohen et al. (2019). The price of stock s_i is influenced by changes in scarcity of all stocks because of substitution and complementarity relationships as show in Eq (5). Rouhi Rad et al. (2021) show it is possible to econometrically recover complementarity relationships between produced and natural capital, Yun et al. (2017) estimate complementarity relationships among stock of natural capital, and Blachly et al. (2024) provide evidence of complementarities in watershed services.

Wealth accounting approaches that use superlative indices to measure changes in real wealth allow for varying patterns of substitution, including limited substitutability. For example, computing income or change in wealth from price-quantity data is the geometric mean of Laspeyres and Paasche indexes,

$$(8) \qquad \Delta\mathcal{W} = \left[\left(\sum_i \frac{p_i(\boldsymbol{s_1})s_{i,1}}{p_i(\boldsymbol{s_1})s_{i,0}} \sum_i \frac{p_i(\boldsymbol{s_0})s_{i,1}}{p_i(\boldsymbol{s_0})s_{i,0}}\right)^{\frac{1}{2}} - 1\right]\mathcal{W}_0$$

where Eq (5) is used for the prices at time zero and time one. Expression (8) is neither linear in prices nor quantities. It depends on the full set of cross partial derivatives, which means that the change in real wealth reflects changes in the substitution or complementarity relationships implied by 1) the set of observed stock shifts and 2) the fact that prices of any one stock can in principle depend on all other stocks.

To illustrate how changes in substitution affect measures of change in real wealth, let the stock s_i become more substitutable with the numeraire good over the shift from period zero to period one and compare it to the counterfactual drawn in Figure 2. An increase (decrease) in substitutability implies an increase (decrease) in the substitution effect, which, all else equal reduces the income effect and leads to a smaller equivalent variation or change in real wealth. Figure 2 can be redrawn one of two ways to illustrate this change (Fig 3, new elements shown in gray and using the $'$ embellishment). The location of point C changes to C' in both cases and the hypothetical constant relative price budget curve is closer to the original curve, than in the base case. It is possible to observe the same price-quantity pair, B, in period one as in the base case (Fig 3, panel A). Imposing an increase in substitutability with numeraire implies a price change elsewhere in the system (though perhaps this goes unmeasured). This increase in the elasticity of substitution causes the indifference curve to bow in, so that the equivalent variation or change in wealth is smaller. This happens because the increase in the quantity of stock s_i must be associated with a greater substitution effect. Alternatively, the location of B could change leading to B' (Fig 3, panel B), implying a higher price for stock s_i relative to the base case in Figure 2 or that the price of stock s_i falls by less, also leading to C' closer to the origin and a smaller equivalent variation or change in wealth. Effects would go in the opposite direction for declines in substitutability or increases in complementarity. For example, an increase in complementarity associated with a stock decline would reveal a larger decline in wealth relative to the same shift in quantity and prices, but with no change in substitutability.

Panel A of Figure 3 reveals the limits of a superlative index approach and importance of complete accounting. The superlative index is a second order approximation to the equivalent variation but relies on the stocks measured. So, if there is no difference between the prices and quantities of the goods with lower or greater substitution, then the superlative index will miss the change in wealth or equivalent variation because of the change in substitutability. Panel B, however, shows how the price changes reflect the changing substitution, and how that maps into the change in equivalent variation or the change in real wealth. Imputing prices with equation (5) provides the price change effects needed to accurately measure the change in wealth or equivalent variation even when substitution is changing.

4.3 Link between change and wealth and social benefit-cost analysis

National accounts data are seldom used in regulatory or programmatic benefit-cost analysis. Many forms of benefit-cost analysis require prospective simulation of a world with and without the regulatory or programmatic intervention. While one could prospectively simulate macro-dynamic changes and use forecasted changes in a stylized national accounts summary with and without a policy change, this is generally beyond the scope of the standard practice of assembling national accounts.

On the other hand, national accounts enable a sort of *ex post* assessment of progress. Specifically, measuring real wealth enables the thought of experiment asking simply how much more wealth does society have in period one compared to period zero if for a set of constant relative prices. There is no specific (micro) policy change, rather this provides a headline statistic measuring the change in real wealth that encompasses the set of all (macro) changes stemming from the economic program followed between periods zero and one, which complicates benefit-cost analysis that ideally looks for a clean, precise counterfactual with minimal extraneous variation confounding the interpretation of the resulting analysis.

The core difference between a prospective (e.g., regulatory) and *ex post* (e.g., national accounting) analysis is the driver of $p_1(\boldsymbol{s}(1))$. In a typical micro-oriented prospective benefit-cost analysis, this would come from simulating a world with a policy change and comparing to $p_0(\boldsymbol{s}(0))$ as the world without the policy change. In a national accounting or sustainability assessment application, $p_1(\boldsymbol{s}(1))$ would be observed prices or come from imputed prices based on observed actions.

It is only in this narrow sense that a change in real wealth could be interpreted as a kind of "social benefit-cost analysis" or rather ex post assessment of observed macroeconomic dynamics. This sort of assessment is important, and there is no reason that change in real wealth could not be used in a similar prospective fashion as simulations of GDP growth – but that is not usually what is meant when benefit-cost analysis is discussed in most regulatory policy applications, unless the breadth of the policy has a sufficiently macroeconomic reach (e.g., monetary policy). Still, the connections between benefit-cost analysis and wealth accounting highlight the path to aligning methods for imputing prices, based on methods developed in the context of regulatory and programmatic benefit-cost analysis.

4.4 Nonmarket valuations and measurement for natural capital accounts

The theoretical development in Section 3 starts by setting aside the issue of what counts as contributing to value and focuses on the theoretical concept of what is value once a service flow is considered to matter. Section 3 goes on to show that a change in real wealth, which is Hicksian income, is the *real* change in the sum of the exchange values. The real change is computed using the standard method of superlative indices used in the national accounts. Furthermore, it illustrates the notion that it is feasible to convert all assets to cash savings at observed prices, and that creates the only necessary hypothetical for the analysis.

Section 3 also shows that change in real wealth can be illustrated as the vertical shift in the numeraire asset (i.e., cash-equivalent savings in real terms), which is the welfare concept of equivalent variation commonly employed in environmental economics. It is important to be clear that the "counterfactual" in this case is the assumption that relative prices would have been as they were in the prior period. There is a sizable literature showing under what conditions change in consumer surplus approximates equivalent variation (Dynan and Sheiner, 2018; Phaneuf and Requate, 2017; Willig, 1976).

The challenges in using equivalent variation or change in consumer surplus in national accounts stem from understanding where they can be used, not if they could be used. There are at least three challenges that this paper helps clarify. First, there is the issue that a measure would need to be in terms of a real, as opposed to nominal, change in consumer surplus or equivalent variation. Second, it is critical to keep track of changes in the numeraire and other goods. This works against using change in consumer surplus but is not an issue (with careful accounting) for equivalent variation.[25] The third, and perhaps the greatest challenge, is that change in consumer surplus or equivalent variation requires the careful netting out of intermediate goods and the careful accounting of when flows are shifted from intermediate goods to final services that add to income and hence change wealth. This is ideally what national accounts endeavor to do. Indeed, the greatest impediment of using equivalent variation measures directly in change-in-wealth measures is taxonomic alignment with the accounting scheme. It is not necessarily a misalignment in the concept of real value, as we have shown.

Change in consumer surplus or equivalent variation could be used as part of a change-in-real-wealth measure, but they cannot be substituted for nominal exchange values. This distinction is important. Exchange values are meaningful for finding the total wealth normalization in nominal terms, which is the hypothetical value if all assets could be converted to cash at constant prices.[26] It is convenient to use the total to decompose to prices and quantities that enable the computation of a percent change that does not depend on the normalization.

[25] And change in consumer surplus may still be a reasonable approximation.

[26] The hypothetical nature of this quantity applies to standard market assets as well as nonmarket assets.

An important take away is that prices and quantities, rather than values, are the elements needed to measure real net income or change in real wealth. Consider a welfare-based analysis of demand for asset s_i in period one. The economists conducting a benefit-cost analysis might report consumer surplus associated with some change away from $s_i(1)$, or they might report the equivalent variation associated with this change. However, if they report either the ordinary Marshallian demand curve or the Hicksian demand curve, then point B is identified (Figure 2, lower panel). The exchange value in period one is known – even if that was not the objective of the non-market valuation study. This price-quantity pair could be used to assess the change between this period and the next period if the measure were repeated in the next period. In theory, this opens the door for national accountants assembling wealth accounts to make use of a substantial amount of nonmarket valuation research that focuses on demand estimation (e.g., Klaiber and Phaneuf, 2010; Phaneuf et al., 2009) and is clear about alignment with observed conditions, particularly if the nonmarket valuation approach 1) focuses on an asset, good, or service within the accounting boundaries of the national accounts, and 2) they are measuring the *change* in welfare values from some relevant positive price-quantity pair benchmark, i.e., *not* the *total* welfare. Indeed, environmental economists are starting to deploy these methods at the scale necessary for national accounts (Bieri et al., 2023; Chan and Wichman, 2022), though seldom exactly aligned with the needs of national accounts.

There are, however, some caveats and qualifying circumstances where the scope of the national accounts may not align with nonmarket valuation techniques employed in environmental economics or benefit-cost analysis. The first relates to W, and whether the services flows used to locate the demand curve in the quantity-price space are within the accounts deliberative boundaries for what counts in W per national accounting taxonomies. There are some clear cases where the services are within the national accounting boundaries, like valuation of land, subsoil assets, or timber. However, there may be concern that misattribution will lead to double counting. For example, hedonic pricing models are used to estimate value related to proximity to forests or beaches for any set of services (e.g., recreation or flood control). Yet, one concern arises if the value of housing is captured in the accounts, which already capitalizes some value of natural amenities in land value. This seems reconcilable as part of a reattribution, provided that these values can be decoupled. In other cases, it is possible that important services are outside the current accounting boundaries. Consider an air quality improvement that enhances labor productivity. It is likely that increased quality of home-cooked meals or more efficient machines similarly improve labor productivity. The question is whether the air quality change is more like the home cooked meal improvement or the machine improvement, because the former is outside the accounting boundary. Finally, there is the issue of economic ownership. Consider Manning and Ando's (2022) results linking wild bat populations to agricultural output. Declines in bats generate a negative effect on farm income through a mix of declining production and increased pesticide cost. However, the bats are not owned by the farm and may be imperfectly managed on behalf of the farmers by a state wildlife agency. This raises questions of where to put the asset value in a set of accounts, rather than questioning whether the bats are a valuable asset. This shows that the core of the issue is not theoretical, nor is it technical, and the data challenges are rapidly resolving. The core of the challenge is the need for nonmarket valuation studies that align with deliberative accounting boundaries, such as current SNA boundaries or other extended boundaries a statistical office may choose to adopt,[27] that may not fully respect consumer sovereignty. Creel et al. (2024) illustrate how this may work by partitioning the source of travel costs in a travel cost model. For this to happen, it is important that there is a reasonable taxonomy with which to align non-market valuation studies.

Scale and coverage creates one final challenge with accepting much of the nonmarket valuation literature, "off the shelf," into national wealth accounts. Imputed prices are location dependent, depending on biophysical conditions and local economic programs (Addicott and Fenichel, 2019). Beyond that if only a small amount of natural capital is measured there is a question as to whether elasticities are constant as the estimates are extended out of the range of the data. This means there is need for more systematic nonmarket valuation studies that capture local variation while providing consistent estimates for national aggregation, focusing on 1) the marginal changes to welfare rather than total welfare and 2) real values rather than nominal. A core idea from this paper is to suggest that, because of equivalent variation and the alignment of the change in welfare with changes in real value, a subset of welfare analysis may already be useful for national accountants *and* a subset of real national accounts values may be useful for economists interested in measuring changes in welfare. That is, national statistical offices may want to consider using special cases of nonmarket valuation techniques as a robustness test for measuring changes in real values, particularly for natural assets where there is a developed literature and established conventions on these techniques. On the other side of this coin, environmental economics and the broader benefit-cost analysis literature, which employ some form of welfare analysis, may want to consider how to better exploit real wealth and real income measures in the national accounts (at least for ex-post assessment), even if it appears to be missing the explicit measurement of "triangles" known as consumer surplus.

This section shows that the theory and methods exist to do the valuation needed for natural capital in wealth accounts. The challenges it raises are surmountable. They are organizational in nature not intellectual in nature.

[27] The U.S. strategy recommends two additional boundaries be considered beyond the SNA boundaries (OSTP et al., 2023).

5. Conclusion

This article addresses three questions that seemingly have slowed progress on developing and institutionalizing wealth accounts and change-in-wealth measures of sustainability. First, the paper clarifies the connection between exchange value and welfare value concepts, showing that in principle welfare theory and national accounting traditions are compatible in measuring changes in real value but require clarity on concepts and appreciation for accounting boundaries (and why they exist). In so doing, it also addresses how measurements necessary for change-in-wealth measures relate to standard national economic accounts measurements.

Currently, the international guidance and statistical standards for non-financial balance sheets include cultivated biological resources (e.g., plantations and livestock), land, mineral and energy reserves, non-cultivated biological resources (e.g., managed wild forests and managed wild commercial fish stocks in exclusive economic zones), water resources, and "other natural resources." However, few countries are currently reporting these components on their non-financial balance sheets. Still, these categories are likely not sufficiently complete to align with what individuals would include in their own wealth assessment. The international statistics community has also standardized the System of Environmental Economic Accounting (SEEA), and at least 90 countries have begun the process of implementing some of these additional measures.[28] However, an important gap in the international statistical guidance is that SEEA provides limited guidance on the difficult question about how precisely non-market valuation techniques should be deployed for national economic accounting, in large part reflecting a general inconsistency in the understanding of the connection between economic theory and national accounting as elaborated in this paper. This is reflected by the fact that only 15 countries have developed any form of SEEA monetary account for water, 6 for land, and 1 for aquatic resources.[29] The United States National Strategy for Natural Capital Accounts and Environmental-Economic Statistics (OSTP et al., 2023) highlights the need to establish valuation guidance for implementing a national natural capital accounting strategy. We show that the core issues are not in the types of methodology, but in what counts, in taxonomies and classification systems, in coverage, and in clarifying what value is being measured (i.e., real, nominal, or both; and, marginal or total).

This article also addresses how the change-in-real-wealth approach to sustainability assessment takes a practical approach of measuring substitutability rather than taking normative stances about strong and weak sustainability ex ante. Index numbers do not just address the issue of comparisons across periods because of inflation, but they also address concerns about general changes in prices that result in substitution. They reveal that all else equal the loss of substitution possibilities yields larger losses in

[28] https://seea.un.org/content/2023-global-assessment-results
[29] https://seea.un.org/content/2023-global-assessment-results

wealth. For substitutability to increase wealth, it must come from shocks like actual innovation, discovery, or institutional change. Furthermore, change-in-wealth metrics do not impose assumptions about sustainability, rather they reflect the ex post management of all assets. The greater concern when it comes to substitutability is if society truly acts as though complements are actually substitutes, but this is just a special case of society getting the prices "wrong." As Fenichel and Abbott (2014) point out, the power of well-compiled wealth accounts is to drive institutional change that improves asset management, and perhaps corrects prices, so that we see the sustainability improvements when they happen.

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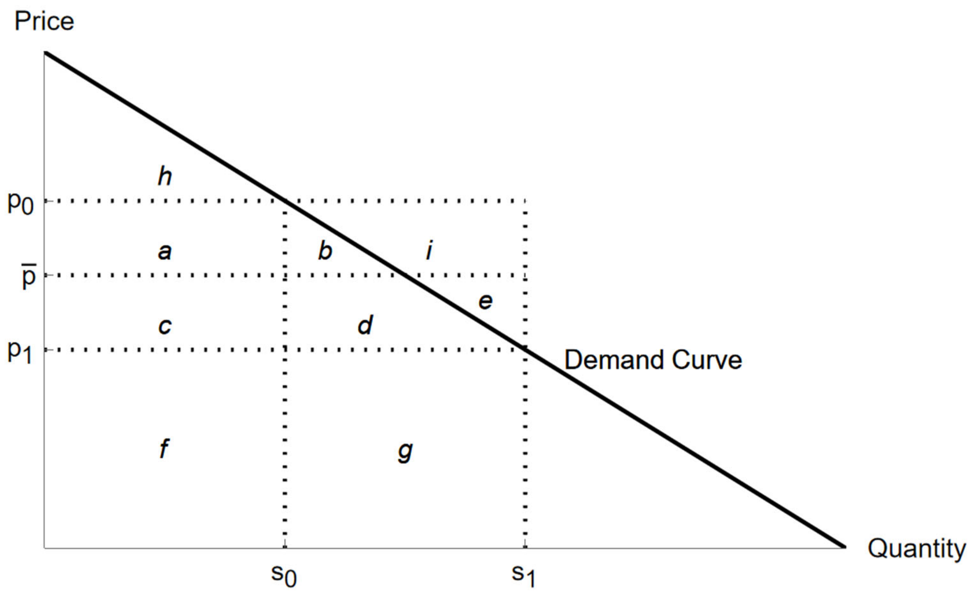

Figure 1. Illustration of constant price concept.

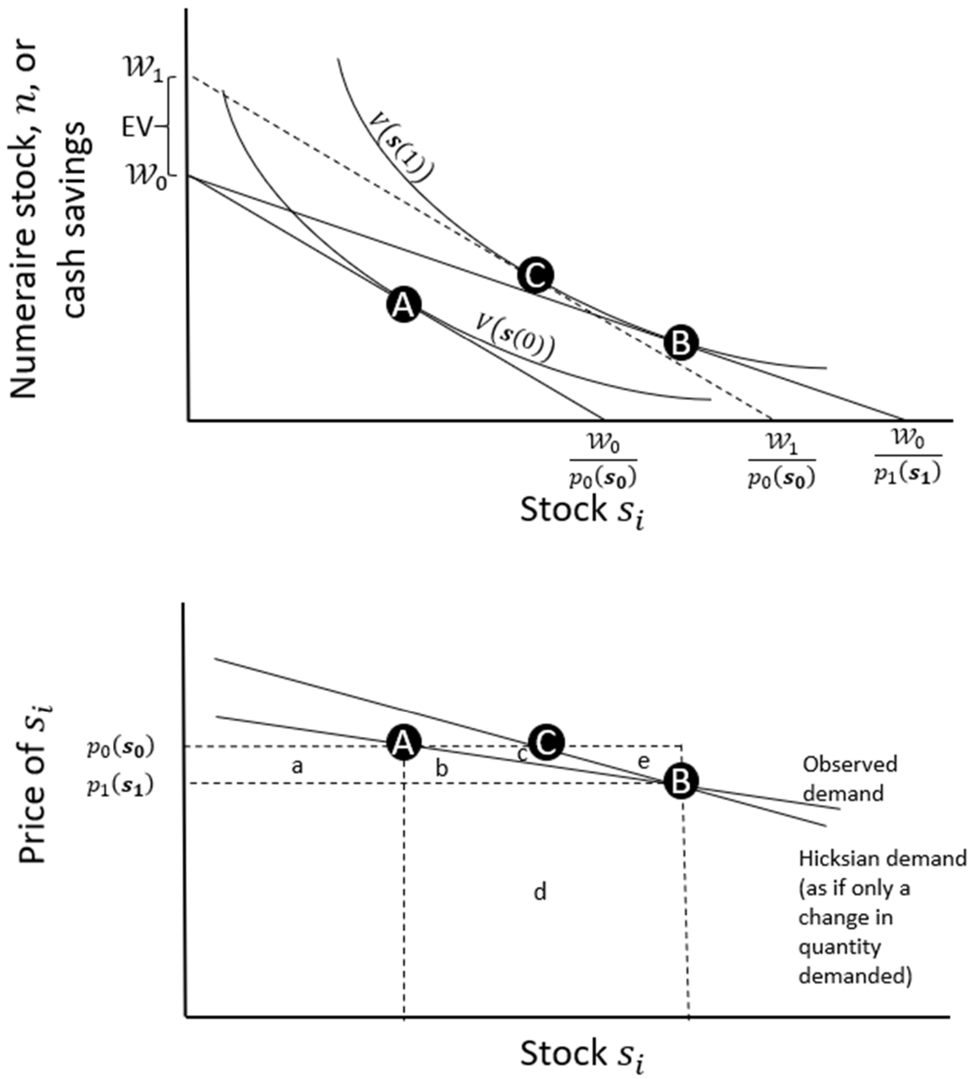

Figure 2. Connection between equivalent variation and change in wealth.

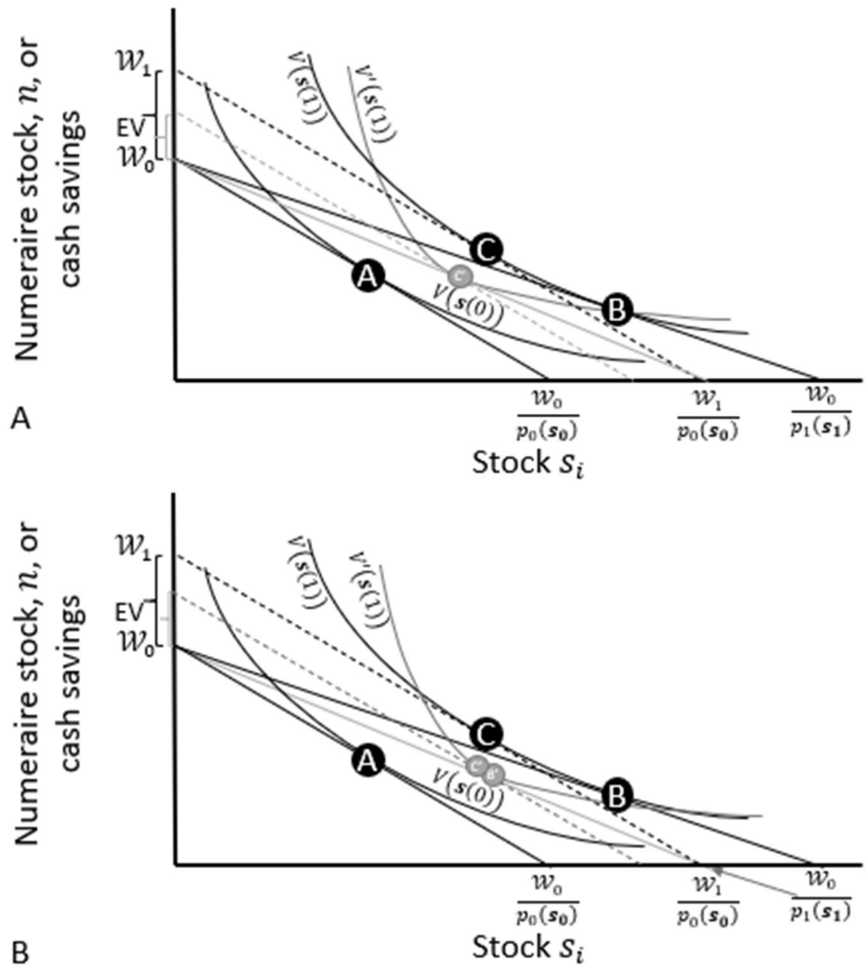

Figure 3. Illustration of how equivalent variation and superlative indices capture changes in substitution.