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A framework for the valuation of ecosystem assets

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Working group 5: Valuation and accounting treatments

Discussion paper 5.2: *A Framework for the valuation of ecosystem assets*

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Executive Summary

Developing a complete set of national environmental or ecosystem accounts requires a capital account and a balance sheet. Core to compiling these accounts, is the development of physical and monetary measures, where stocks of natural resources and ecosystem assets are valued at appropriate accounting prices. This paper focuses on pricing assets and measuring changes in the value of assets in the context of ecosystem accounting. The core concepts of asset valuation and price in (welfare) economics and national accounting largely are consistent. In theory then, there is general agreement that the value of an asset is the net present value of the income flows that it provides and the price, or marginal value, of an asset is the change in net present value that an additional unit of that asset provides.

From this starting point however, various perspectives/understandings on its application have emerged, particularly concerning the application to the valuation of environmental assets. This paper examines these perspectives in considerable detail, from an economic theory perspective, with the particular aim of bridging to national accountants understanding of the measurement issues.

Key findings from the paper are that:

- The core value concepts in economics and national accounts are based on exchange, and both reflect welfare considerations.
- Changes in welfare and changes in the values recorded in national accounting are approximately connected through index number theory if the world is in full general equilibrium and all changes are endogenous. Therefore, assumptions about partial vs general equilibrium matter as do institutional assumptions. This is because the price-quantity pair is a limited information set with respect to welfare. Nevertheless, this connection to index number theory provides a path forward for making greater use of welfare-based environmental economics in national accounts – though extreme care must be taken to isolate the price-quantity pairs not the welfare measures for national accounting purposes.
- National accountants and economists should be able to agree on prices and quantities and approximately on changes in value between price-quantity pairs. There is nothing apparent in welfare economics or accounting theory that prevents this.
- National accounts and economists struggle when ecosystem assets are poorly defined, but are very capable of considering price when the physical units of the asset are well defined.
- A major point of diversion between economics and national accounts involves the production, asset, and income boundaries. In the System of National Accounts, the boundaries are based in general principles but in many instances are determined by convention, often for practical reasons. The boundaries are especially challenging to establish when non-market interactions influence general equilibrium price changes, which is the common situation for environmental assets.
- A commonality that impairs progress is that economists and national accountants have failed to take advantage of the potential to describe an underlying account structure and balance sheet that supports measurement of different aggregates. This is not as simple as it sounds however because a common set of prices should be supported in the accounts, but when these prices are imputed, the measurement boundaries must be agreed initially – at least with respect to substitution opportunities influencing prices.

Overall, It is hoped that the material can serve to support a richer discussion of environmental valuation questions by encouraging greater among experts from different areas.

1. Introduction

In the framework of the SEEA Experimental Ecosystem Accounting (SEEA EEA), ecosystem assets are a core element reflecting the conceptual unit that supplies a critical input into the production of ecosystem services and the statistical representation of ecosystems as defined through ecology. Understanding the capacity of ecosystem assets to supply ecosystem services into the future is a key motivation for ecosystem accounting, allowing, as it does, a pathway to the discussion of sustainability and resilience. Indeed, Kuznets (1973) emphasized that national accounts based measures of economic growth are not relevant for "countries [that] can provide increasing income to their populations because they happen to possess a [natural] resource" and that it is important to address "the so far hidden but clearly important costs… [of] pollution."

Within this context, there is a long-standing ambition to understand the changing economic value of ecosystems and other non-market contributors to human wellbeing (reviewed by Fleurbaey and Blanchet 2013). At a national level, this has been driven at times by a desire to adjust measures of GDP to recognize the cost of using up natural capital and at other times by the aim of establishing a broader measure of changes in national wealth that encompasses all types of capital including natural capital. At a finer scale, work has focused on the relative contributions of ecosystems and components of natural capital to production and consumption. This discussion paper focuses on a description of the relevant economic and accounting concepts and theory that support measurement in this area, i.e. on the valuation of ecosystem assets.

One motivation for the paper is that the dialogue between national accounts and the environmental valuation community has not been extensive in recent times. The SEEA EEA revision process thus provides an opportunity to make connections among the various experts and to reach an agreed understanding of the appropriate pathways towards valuation for SEEA EEA purposes. As a consequence, the audience for this paper is both the national accounting and SEEA communities who are familiar with national accounts approaches to valuation and the environmental economics and related communities who are familiar with the literature and practice concerning the valuation of environmental assets. While targeting both audiences, the paper has been written largely from the perspective of economic theory. Effort has been made to bridge differences in the use of terms and concepts between economics and national accounting but it is likely that not all issues of language and description have been resolved.

The logic of the paper is to first describe core concepts in the valuation of assets generally. This then provides a clear basis for the discussion of valuation of natural and ecosystem assets[1] in particular. Along the way, there is discussion of a range of important issues of terminology and understanding in this space. The broad objective is to provide a commonly agreed language and concepts for the discussion of the valuation of ecosystem assets and for making choices on measurement boundaries and associated treatments in the revision of the SEEA EEA. It is evident in a number of cases that further discussion is required to fully reconcile the economic and national accounting perspectives. The paper provides a description of the differences in perspective where appropriate.

This discussion paper sits alongside two other discussion papers that are being prepared in parallel. The first concerns the valuation of individual ecosystem services including the connection between so-called exchange and welfare values (a topic also discussed here). The second concerns options for the incorporation of ecosystem values into standard accounting formats, in particular the appropriate accounting for ecosystem degradation. We have made some effort to connect with these discussion papers, but further consideration of issues of consistency will be required following ongoing feedback and discussion.

[1] The term "natural and ecosystem assets" is used throughout this paper refer to the potential targets of asset valuation. The reference to ecosystem assets is the same as envisaged in the SEEA EEA. The reference to natural assets is similar to the concept of natural resources and individual environmental assets as envisaged in the SEEA Central Framework but more specifically in this paper it relates to the components of ecosystems that variously contribute to ecosystem functioning and hence the flow of ecosystem services.

2. Core concepts in valuation in the valuation of assets

2.1 The welfare context

In economics, the value of an asset is the net present value of income associated with the asset, and the price of that asset is change in the net present value with respect to a change in the stock of the asset (Jorgenson 1963; Hulten 2006). Fenichel and Abbott (2014) and Fenichel, Abbott, and Yun (2018) apply exactly this definition to natural capital and ecosystems. However, on its own the definition is of little use without first clarifying exactly what is meant by the concepts of value and income, and to whom they relate.

Welfare economics is what economists, especially those dealing with environmental valuation, think underpins the concept of value (Freeman 2003; Phaneuf and Requate 2017). This concept of value is sometimes called "welfare value" in discussion of the United Nation's System of Environment-Economic Accounting (SEEA). The crispest operational definition of *individual* welfare appears to be Hicks's (1939) statement that *income* is the "maximum amount of money which an individual can spend this week, and still expect to be able to spend the same amount in real terms in each ensuing week." Modern notions have generalized Hick's definition to replace the first "spend" with consume and second spend with "benefit from" – shifting the focus to utility because non-market services are also important (Sefton and Weale 2006).

Heal (1998) states that there are two income concepts discussed in the literature on national income and the environment; (i) the Hicksian income concept and (ii) attempts to construct an index number with welfare significance (see Diewert 1992).[2] In these "real income" approaches, assets are durable goods that support the generation of opportunities.[3] The value of an asset is equal to the net present value of the expected income that that asset generates (Hulten 2006). Increases in assets enable an increase in the level of consumption that could be maintained in perpetuity.

In the context of national accounts, care is often taken to express that "value" is not welfare related. As will be discussed below it is certainly the case that national accounts values exclude measures of consumer surplus.[4] However, more broadly since welfare theory is the dominant normative theory of economic analysis and evaluation and the main objective of the accounts "is to provide a comprehensive conceptual and accounting framework that can be used to create a macroeconomic database suitable for analysis and evaluating the performance of an economy" (European Commission et al. 2009), it is important to recognise that the use of the accounts relies on the application of welfare theory for their interpretation. Indeed, this point is acknowledged in the 2008 SNA (European Commission et al. 2009).

Within this broader framing of the link between national accounts and welfare, Nordhaus and Tobin (1972) argue that the "value" of interest more broadly are the opportunities for "consumption" that may include, non-

[2] Weitzman (2016) also states that, "contemporary national accounting is centered conceptually on what is frequently identified as the 'Hicksian' concept of income." Nordhaus and Tobin (1972) offer a "measures of economic welfare" (MEW) that is essentially gross and net national income using the broadened notion of Hicksian income. They focus on actual as opposed to potential measures, but also consider sustainable measures.

[3] The notation of "real" income is often associated with inflation, which relates to the changing value of money that parallels changes in the money supply. However, it is worth pausing to realize that the money supply changes in large part because of changes in demand for debt associated with changes in the productivity of capital assets and their substitutes (Tobin 1961). Therefore, "real" is more general than inflations concerns alone, it has to do with adjusting for substitution effects associated with the numeraire.

[4] At least this is the claim, however changes in consumer surplus do materialize when index number theory is used to consider changes in value through time. We will illustrate this below. This is important because the welfare interpretations are about changes not levels.

consumptive contributions to welfare.[5] Further, it is well established that changes in net national product (NNP) are a first-order approximation of changes in welfare under certain assumptions (Harberger 1971; Weitzman 1976). However, national accounts claim *conventions,* not concepts, lead Gross Domestic Product (GDP) not to track welfare. Overall, it is clear that we must better understand the value concept embodied in the national accounts, which is often called "exchange value."

It is worth clarifying some of the issues around NNP in order to understand the challenges with measuring changes in welfare and what assumptions are require for national accounts to provide approximations. First, there is usually an assumption that net national product equals net national income based on the circular flow model (Hulten 2006). One limitation of the current set of accounts has to do with the production boundary excluding services that only require natural capital inputs and household labour, even if market substitutes exist. This leads to a wedge between production and consumption. Said differently, change in NNP is only a welfare approximation to the extent that all stocks are included and all services flows produced by capital stocks are accounted for – including non-market services. Second, aggregation at a point in time can be done in terms of a numeraire currency, which implies that members of the society's utility functions are linear in the numeraire currency. This is Dasgupta (2007) primary critique of Weitzman's analysis, noting that such an assumption is at odds with household saving behaviour and insensitive to distributional issues. Dasgupta (2007) does offer two alternative welfare "uses" for NNP. First, defining NNP as "the sum of genuine investment and the accounting values of all consumption services (including direct consumption services from the natural environment and 'negative consumptions arising from pollution (Nordhaus and Tobin 1974))," then "the value of consumption services must not exceed NNP" for welfare to be non-declining over time. Alternatively, NNP takes on a welfare significance if it is redefined as "the value of net changes in the flow of consumption services plus the change in the value of net investment."

In bridging between the economics that contributes to valuing ecosystem assets and national accounting, a critical step is to understand what ideas are shared to facilitate cross-fertilization, reduce redundant effort, and engender broad credibility. In particular, to successfully value natural and ecosystem assets, it is important to build common ground between value and price concepts. The value concepts in economics and national accounts are not as divorced as it may seem and understanding the relationships is important for understanding how the environment can and should enter national accounts and how to value natural and ecosystem capital.

2.2 Price and Value

The concepts of value and price are central to economics and national accounting, but while price and value are related, they are not interchangeable concepts. Furthermore, a terminology of "welfare value," "welfare price," and "exchange value," and at times "exchange price" proliferates in conversations of environmental-economic accounting, and some discipline must be brought to bear.

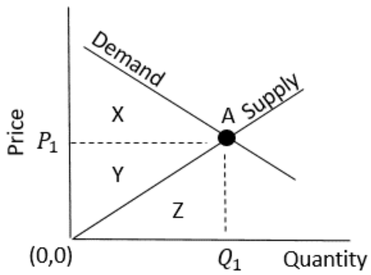
To simplify exposition, we abuse language a bit and define "gross welfare value" as the sum of traditional welfare economics notion of value and cost and "net welfare value" as welfare less costs. The difference is shown in the static representation below (Figure 1) where net welfare value is approximately the sum of consumer (Fig 1 region X) and producer surplus (Fig 1 region Y).[6] The national accounts concept of exchange value could similarly be defined as gross (Fig 1 regions Y and Z) and net (Fig 1 region Y).[7] In any event, concerns about netting out costs (Fig 1, region Z) are not where confusion arises, but including costs in a notion of "welfare value" adds confusion for

[5] Nordhaus and Tobin also emphasis that "An obvious shortcoming of GNP is that it is an index of production, not consumption. The goal of economic activity, after all, is consumption."

[6] Why the sum of consumer and producer surplus is an approximation welfare measure is addressed later in this paper.

[7] Although this may introduce some additional confusion since the aggregate "value added" in the national accounts will comprise producer surplus plus the employee compensation (wages) component of costs.

economists without also introducing the concept of "gross welfare value." We simply state at this point that the producer surplus is part of the "exchange (or transaction) value" and "welfare value" concepts.



A

Figure 1. Prices, quantities, producer surplus, and consumer surplus.

To further clarify exposition, it will become clear in the following that the concept of exchange is relevant in all valuation and pricing discussions. Thus, to clearly distinguish the national accounts concept of value the term "transaction value" will be applied in this paper.

Exchange is central to the notion of value. The value of something is its worth in the units of something else. For example, the quantity of item B needed to exchange for a quantity of item A. Because value is based on exchange, only the extent of the exchange or change can be valued.

Price is connected to value. For economists, price is the *marginal* value. It is the change in the value of a good (service or asset) as the quantity of that good (service or asset) changes. If there is only a single good (service or asset), then geometrically, value is the area under the price curve between two quantities. Mathematically, a price is the derivative of value with respect to the good being valued. For a change from zero to Q_1 in Figure 1, this is the area X+Y+Z. Value and price are connected through the fundamental theorem of calculus the same way an integral and derivative are connected. The price concept is very helpful and makes valuing small or marginal changes relatively easy. In exchange systems, prices signal scarcity, enable voluntary exchange to act as if there is an "invisible hand." It is in this sense that exchange aligns at all with social objectives.

From a national accounts' perspective, the link between price and value is most commonly understood in a transactional context. Thus, for any given transaction in goods, services or assets, there is a price and a quantity component, with the price most commonly reflected in monetary (numeraire) terms. These values are meaningful from a perspective of understanding the current financial context for economic activity – i.e. how much cash is in the bank – and also are the necessary starting point for the collection of economic data. On the basis of organising data on transaction values, national accounts then use index number theory to decompose changes in transaction values into price and quantity/volume changes. The critical point is the prices inherent in transaction values are based on exchange, and the market illustrated in Figure 1 clears at point A with quantity Q_1 and price P_1.

Welfare prices are exchange prices. What should be clear from Figure 1 (above) is that the price concept in national accounts and the price concept in welfare economics is the same. The 2008 System of National Accounts (European Commission et al. 2009) defines [exchange or transaction] price of a good or service as "the value of one unit of that good or service." This is less precise than the economist's definition that says that the price of a good, service, or asset is the value of the *marginal* unit of the good, service, or asset. The first fundamental theorem of welfare economics states that if contracts are complete then voluntary *exchange* at prices reflecting the marginal value are Pareto efficient. The important point is that a welfare price emerges from the same mechanism as an exchange price.

Another important point is that few national accountants or economists truly believe all exchanges are in the form of complete and competitive contracts (Bowles 2004). Nevertheless, we can relax the strong assumptions in the welfare theorems to allow institutional constraints or a rich set of transaction costs, which would lead to institutionally constrained Pareto efficiency. This implies that prices obtained from non-market valuation focused on welfare analysis under prevailing institutional conditions are exchange prices – at least for the observed quantities considered (we clarify this below). The rest of this section clarifies concepts of value and how additional assumptions or information is used to establish notions of value. We also establish that there is no theoretically consistent notion of total value, despite repeated used of the term. The closest thing to total value is the change in value associate with moving from the current stock to a level of the stock equal to zero. For many forms of natural and ecosystem assets this is not a useful thought experiment.

Considering Figure 2a below, for an observable exchange of a market good, we could observe the quantity exchanged as Q_1 and the market clearing price as P_1 in Figure 1 – we don't actually observe demand curve D_m either. Alternatively, we might observe the area of the rectangle created by points, (0,0), P_1, A, and Q_1, and the quantity Q_1 and use these to infer price P_1. This is the price that would occur after the exchange mechanism established a price for the marginal unit and market cleared. The key assumption is that contracts are competitive and can be established independently. Otherwise additional trades would take place and those additional trades would be intermediate (there would be pure arbitrage rents). Critically, although the price-quantity pair established through an exchange mechanism reveals a price, it does not provide sufficient information to say anything about welfare or transaction values without additional assumptions or information.

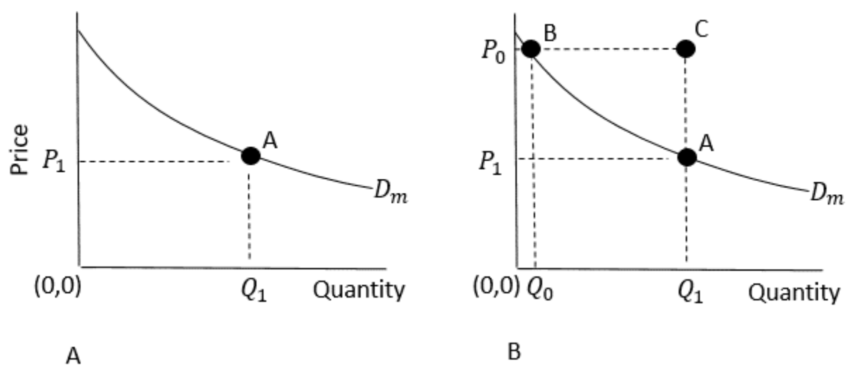


Figure 2. Prices, quantities, and changes in value.

It is incorrect to conclude that transactions values can reveal nothing about welfare values. For example, one possible assumption to interpret transaction values is that all exchanges in a given market are marginal. National accounts, in their decomposition of transaction values into prices and quantities (volumes), almost always make this assumption (Muller, Mendelsohn, and Nordhaus).[8] An observationally equivalent assumption is the price curve is flat. Either way, these *assumptions* impose a notion of value that is $P_1 \times Q_1$. This idea of accounting at constant prices is central to accounting concepts (Nordhaus and Tobin 1972; Dasgupta 2014), and it is approximately the value added to the economy by adding quantity Q_1 to the economy once costs are subtracted – as costs also sit in the rectangle (setting aside labour costs – see footnote 7). Therefore, it is approximately gross total value added – a marginal concept - assuming a large amount of pre-existing Q or many substitutes for Q.

These final assumptions concerning the relative level of Q are critical. To explain further, consider that in some cases, a change from a large quantity to zero can be valued relatively readily, e.g., the change in the quantity of cake in a bakery or even the change in the number of trees on a specific hectare of forest from many to none.

[8] This is part of the reason that they have to be periodically re-indexed, and have proven a less useful tool for long-term assessment or planning.

However, in other cases, the value of a change, e.g., removing all the water on the planet, are nonsensical changes to consider, because there is no sensible notion that the alternative state of "zero" water can be a point of comparison for the estimation of value. This is despite the fact that an estimate may be made of the current volume (quantity) of water. Nordhaus (2006) calls this the problem of establishing zero. More broadly this should highlight that establishing a total economic value of the earth's environment is not a meaningful endeavour (see for example Toman's (1998) critique of Costanza et al.'s (1997) valuation of the world's ecosystems).[9]

Further, consider that, tautologically, total value must equal the value associated with changing from a level of zero Q to Q_1. Transaction value is thus only a total value if prices do not reflect scarcity. Consider a change from zero Q to Q_1 that happens over two periods. In the period quantity Q_0 is introduced, the market clears at P_0 (Fig 2b). In the next period the quantity of Q is expanded to Q_1, and the market clears at P_1. However, generally $P_0 Q_0 + P_1(Q_1 - Q_0) \neq P_1 Q_1$, despite the observed transactions being on the left-hand-side, the gross transaction value is on the right-hand-side. An alternative is to think about losing quantity Q_1. The loss could only be valued at $P_1 Q_1$, which is not credible for market goods. The price of a new car times the total number of cars in the world is not what people would exchange to avoid losing all cars.

Rather than using only assumptions, it may be possible to observe multiple similar markets across space or over time, where quantities and prices vary, and infer a market demand curve D_m (setting aside econometric identification challenges). When the market demand curve slopes down, price measures scarcity (Fig 2b). An interpretation of the market demand curve is as a sequence of transactions as scarcity is relaxed. The area under the market demand curve D_m between two values of Q and above P_1 is the change in consumer surplus, e.g., between Q_0 and Q_1 (Fig 1b). The entire area under the demand curve is the sum of consumer surplus, producer surplus, and costs. This is an approximation of change in gross welfare value, or if costs are removed, then it is an approximation to change in net welfare value (the traditional notional of welfare value). However, this is not truly change in welfare. The reason why this is an approximation is that consumer surplus "mischaracterizes" substitution effects that are welfare agnostic and consumer surplus measure cannot be derived from a consumer's underlying utility function (Freeman 2003).[10]

Nevertheless, consumer surplus reasonably approximates consumer welfare measures such as compensating and equivalent variation (Willig 1976). The preferred welfare measures of compensating and equivalent variation require substantially more information and depend on the individual's right to buy or sell, the reference level for the change, and information from the utility function (Freeman 2003). This leads to consumer surplus remaining an important measure. However, it is helpful to consider welfare analysis to understand the information about value in prices and quantities. The appropriate demand curve for welfare analysis is the Hicks compensated demand curve. The Hicks compensated demand and Marshallian market demand curves cross in an *ex post* assessment associated with realized welfare changes measured as equivalent variation (Freeman 2003), which would be most appropriate for *ex post* accounting. We make this point to emphasize that welfare prices and exchange prices must be the same thing because the market and Hicks compensated demand curves must cross at the observed market equilibrium. Taking these ideas and placing them in a national accounting context, Harberger (1971) points out that change in net national income and net national product are first-order welfare approximations under certain assumptions (previously discussed), while change in consumer surplus is a second-order approximation.

An important insight is understanding the point of divergence between national accounts and benefit-cost analysis, which is what most welfare analysis is constructed for. In a static analytical benefit-cost context, a common approach to assessing gross or net welfare value is to posit an alternative institutional arrangement, perhaps a perfect market or one without certain externalities. The object of benefit-cost analysis is to ask what are

[9] Toman call's Costanza's estimate a "serious underestimate of infinity." The idea is that presumably humanity would exchange nearly everything not to lose planet earth.

[10] These issues led to a large literature in the early 1970s. Often, Willig's 1976 paper is considered the one that resolved confusion about consumer surplus and welfare measures.

the welfare gains of shifting to the proposed, but hypothetical, alternative. The purpose of estimating the demand curve is to find the area under it, but to be credible must be a change from the current conditions. Therefore, the demand curve must go through point A. Since much welfare economics appears to involve the valuation of externalities along these lines, there seems a basis to consider that national accounts and welfare economics have little in common. National accounts are seldom concerned with this sort of analysis. However, while the hypothetical shift is not of interest for national accounts, the challenges of identifying point A are common to both welfare economics and national accounts.

Concern about the influence of substitution and changing incomes on changes in value are not unique to economists. Treatment of prices changes requires attention for national accounts measures that focus on only using price and quantities and not additional preference information as evidenced by the use of index theory to adjust prices (Harberger 1971; Diewert 1992; Sefton and Weale 2006). This is because non-marginal changes lead to price changes, and price changes lead to substitutions decisions. This is as true for the change in value of money as it is for the change in value of other forms of capital (see footnote 3). Substitution decisions make it hard to measure a change in value relative to just different choices or substitution. This is why the Laspeyres index over states rises in costs, while the Paasche index underestimates them (Diewert 1998). If prices and quantities change, for example between time 0 and time 1, then $P_1 Q_1 - P_0 Q_0$, where P_t and Q_t are price and quantity at time t, is not meaningful for evaluating changes in value (Fleurbaey and Blanchet 2013). Following Harberger (1971), Fenichel, Abbott, and Yun (2018) advocate that $0.5(P_0 + P_1)(Q_1 - Q_2)$ be used to measure changes in value. This approximation will be exact if the demand curve is linear, or as Diewert (1992) points out that the Slutsky substitution matrix is the same in periods 0 and 1.

Figure 3 shows this where $\bar{P} = 0.5(P_0 + P_1)$. The triangles y and z must represent offsetting errors. Fenichel, Abbott, and Yun (2018) point out that with a demand *curve*, i.e. with a changing Slutsky substitution matrix or different rates of substitution in period zero and one, it is unclear ex-ante the exact convex combination of prices that should be used. Diewert (1992) offers different index adjustments for price-quantity measures that provide meaningful comparison under various assumptions, ultimately preferring the Fisher Ideal Index (geometric mean of prices or quantities rather than arithmetic mean), and the topic is further discussed in Fleurbaey and Blanchet (2013). Once index theory is being used to estimate prices stemming from changes in the transaction value of the numeraire or other substitution effects, then the change in an approximation of gross welfare value may not be all that different from changes in transaction value for gross and net national income. An additional challenge might be whether income and product measures balance. Nevertheless, substantial progress can be made by being clear about the process so that the necessary adjustments are made only once.[11]

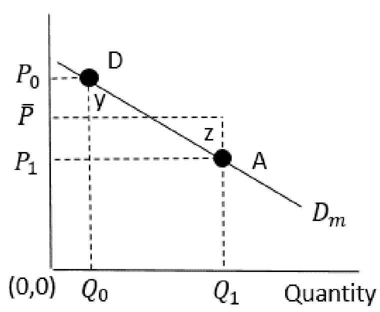


Figure 3. Approximating value changes.

The immediate result is that non-market valuation that matches a price with a quantity can be used to locate point A in Fig1a. However, calculations of changes in welfare cannot to be divided by observed quantities, because:

[11] The order of operations is important when non-linear functions are involved because of Jensen's inequality, which is the mathematical fact that $F(G(x)) \neq G(F(x))$ unless function F and G are linear.

1. The change in question is almost never the total quantity of Q, i.e., non-market demand for the environmental almost never considers zero of the environmental good.
2. For Hicksian welfare analysis, care must be taken to make sure the Hicks compensated demand curve crosses the realized market demand curve at the realized quantity
3. Elasticities may be required to derive an appropriate $\bar{P}$ and then decompose to P_1 and P_2, otherwise assumptions must be used.

Nevertheless, it seems the national accountant and welfare economist should be able to agree on point A, point D, and approximately the change in value between the two up to assumptions (perhaps reflecting different understanding of the curvature of D_m) and data restrictions, without a fundamental disagreement about the notion of change in value.

Perhaps the most straightforward observation is that national accounts most commonly use the term value to refer to stocks and flows recorded in nominal terms whereas economists tend to use the term value to refer to *welfare* changes in real terms. Real terms may seem like a strange term here because real is often used to mean inflation adjusted. However, inflation arises in part because of changes in the mix of capital and substitution opportunities among investment, which impacts money supply, which impacts the value of money in general equilibrium (Tobin 1961). In the current case price changes are occurring because of other sources of capital scarcity and changes in substitution opportunities. In the remainder of this paper the term value will be used to refer to changes in real terms and where a national accounting perspective needs to be introduced the term transaction value (as has been applied here) or "nominal terms" will be used.

3. Valuing and pricing assets

3.1 Applying the concept of value for assets

We now turn to applying the concepts from the previous section to assets. Like goods and services, assets can be exchanged at a point in time between two parties, and this is what is observable in the market. However, assets (capital) are also a means for allowing exchange through time. A unique feature of capital is its durability that enables opportunities to be passed from one time period to next (Scott 1973; Fisher 1906). Dasgupta (2007, p.142) points out that, "economists observe that to say someone is accumulating capital is to suggest that they are sacrificing something now for future benefit." This implies a form of *intertemporal* exchange or exchange through time. An exchange through time might sound complicated, but anytime one forgoes consumption in the present for future benefit that is an exchange between present and future versions of one's self.

Hulten (2006) is precise, arguing if an expenditure is made in the current period to increase future consumption or to prevent a decrease in future consumption, then economic theory is unambiguous – the expenditure should be treated as an addition to savings (an investment), which is an addition to assets.[12] Jorgenson (1963), Tobin (1967), Varian (1992), and Hulten (2006) all emphasis that in the context of assets the appropriate value is the present value of net revenue or more generally the income generated by the asset, which relies on the durable nature of assets. Recognizing that price is a marginal concept, the price of an asset is therefore the change in the present value of income from a change in underlying stock.

Nordhaus (2006) emphasizes that credible national accounts need to include "a full set of current and capital accounts along with the accounts linking the current and capital accounts." In the SEEA context, these linking accounts are likely extent, capacity, or condition "accounts."[13] These are important for assets not exchanged in markets, because the assumption in a market is that prices reflect condition or capacity. Therefore, this additional

[12] Changes in savings and demand for capital also influence the value of money.

[13] The term account is used loosely in this sentence. These extent, condition, or capacity records are more about structured information.

information is needed to impute prices for non-marketed assets. Formally then, the price of an asset is the marginal (or incremental) value associated with a change in the underlying stock (extent, capacity, or condition), often experienced along with a change in time.

Concerning terminology, many modifiers have been put in front of price when discussing the value of non-financial assets when markets do not exist. Exchange price (Obst, Hein, and Edens 2016) is used to indicate that the price comes from an actual or *as if* exchange process. Accounting price (Dasgupta and Maler 2000; Muller, Mendelsohn, and Nordhaus 2011) is used to indicate the price is the appropriate marginal valuation for (national) accounting purposes. Shadow price is commonly used in natural resource economics to indicate that the price is not observed through the market. However, Fenichel, Abbott, and Yun (2018) point out that shadow price is often, though not always, associated with an optimal allocation program, even though it need not be. They suggest using further modifiers such as "revealed" to indicate when a measured shadow price is conditional on prevailing institutions and optimal shadow price to indicate a price under a hypothetical, optimizing economy. In the prior sections, we showed that in realized terms these price concepts all align.[14] Given the prior discussion, in the following sections the term used in situations of actual institutional context is simply "price", unless clarification is required.

3.2 Institutional arrangements

Before turning to the various components of the present value calculation, a short discussion on institutional arrangements is appropriate. For the purposes of the SEEA, and in line with national accounts more generally, the focus is on valuation of natural and ecosystem assets that are (i) not exchanged in markets or (ii) are exchanged in markets that are believed to be highly distorted.[15]

With respect to (i), the measurement focus must turn to using rates of intertemporal exchange (i.e. changes in net present value of real incomes) to impute asset prices. Hulten (2006) makes the point that imputation of prices for assets not traded in the market is only a challenge of implementation, not of basic design. Fama and French (2004) explain that for estimating the price of financial assets traded in the market, a full measure of present value is unnecessary if correlations with assets of revealed value are known. However, Svensson and Werner (1993) show that credible forecasts of the present value increase in importance when assets are not easily traded in markets, which is the situation for many natural or ecosystem assets.

With respect to (ii), economists often find it helpful to assume that an economy optimizes net revenue or real income (e.g., Asheim 2000; Weitzman 1976; Sefton and Weale 2006). Optimization is a useful device for developing theory. However, economic theory can usefully be extended beyond these models, which are intended to produce intuition and potential benchmarks (Bowles 2004; Dasgupta 2007). Indeed, Arrow et al. (2004) and others have focused on measuring changes in wealth as approximations to changes in welfare in economies rife with externalities and market imperfections.

Consequently, the focus here is on the case where the institutional arrangements that guide resource management, often called the economic program or resource allocation mechanism (Dasgupta and Maler 2000; Fenichel and Hashida 2019) are held constant. This may not represent the full opportunity cost of using or losing resources today, if there is the possibility of "better" management in the future.[16] If management institutions are expected to deteriorate, then current assessment could overestimate the opportunity cost of saving resources for

[14] Of course, different institutions and assumptions lead to different prices. The price revealed in an efficient economy will be different than the price revealed in an economy with externalities.

[15] Our focus is on assets. In the case of assets, there can be substantial value even in highly distorted markets, because actions can be observed that influence savings and consumption decisions. These behaviors may imply valuable non-market services. However, how to address services is interconnected with the consumption/ income boundary.

[16] One solution to this challenge is to consider the future management probabilistically.

the future. Consequently, the focus is on revealed rather than optimal shadow prices. This is in line with the types of data available from markets, and hence with national accounts valuation principles; with the caveat that markets themselves are also a way of conveying information (Milgrom and Stokey 1982). An alternative assumption is that exogenous change institutions exist. In the example below we show that assumptions about institutional change have large effects on understanding non-marginal changes in value.

3.3 The Context of Capital

Present value of net revenue or income depends on many factors, and the concept of income (which was touched on briefly in the introduction) requires greater explanation, which we take on momentarily. The following sections consider a few of the other factors that influence the present value of income. Jorgenson (1963) and Hulten (2006) emphasizes the importance of (i) the "replacement rate" or "depreciation rate" of the asset, (ii) the interest or discount rate, and (iii) the expected price changes (capital gains) to the asset, since all of these influence the incremental present value of expect future income generated by holding an additional unit of the asset.

Fenichel and Abbott (2014) show how traditional asset price theory can be applied to natural assets, and Yun et al. (2017) extend the methodology to an ecosystems context. Fenichel, Abbott, and Yun (2018) provide details of how real income, net depreciation, and discount rates, conditioned on institutional arrangements, can be combined with capital theory to measure asset prices and capital gains (price changes over time) associated with asset. In particular, the authors decompose capital gains, which helps to provide greater economic understanding of complex socio-ecological processes.

In the following sub-sections we summarize and provide a guide to asset pricing in the context of natural and ecosystem assets. In short, income, net depreciation, and discount rates require measurement in order to impute the price (i.e. marginal changes in value) for natural or ecosystem assets or to value non-marginal changes in these assets. The final sub-section discusses why it is important to consider the allocation mechanism.

3.4 Income

The concept of income is complicated and has been debated for over 100 years (Weitzman 2016; Hill and Hill 2003; Sefton and Weale 2006). In his foundational text, Fisher (1906) defined "real income" as "services" and included "the supplementary elements which we found lacking under the head of money-income… for it [real income] recognizes that money is only an intermediary, and seeks to discover the real elements for which that money-income stands."

Fisher's definition emphasizes two key features. First, *services*, which means that intermediate commodities used up in generating services are not counted – i.e. costs to produce the services are not income. If Fisher were writing today, he probably could have said that income is what we have called change in net welfare value (which as noted in the previous section may be approximated by change in real net value added). Second, income does not have to be in monetary form. Krutilla (1967) emphasizes this later point in the context of the environment, writing in the American Economic Review, "when the existence of a grand scenic wonder or a unique and fragile ecosystem is involved, its preservation and continued availability are a significant part of the real income of many individuals." Simply put, these are services too. Fisher's definition preserves the need for the concept of assets, and views income as an "interest like return" on wealth (Weitzman 2016).

A slightly different view is due to Hicks (1939), whose income concept is defined as that amount that can be consumed at a point in time without impeding future consumption or the maximum amount of consumption that holds capital intact (Heal 1998; Hill and Hill 2003).[17] A key challenge, for our purpose, is that Hicks seeks to remove the need for tracking assets altogether (Weitzman 2016).[18] Nevertheless, the Fisher and Hicks notion of income align under what Weitzman (2016) calls, "perfectly-complete accounting," – a case with no double counting, no undercounting, and elimination of costs – the case where all stocks and services are included whether exchanged in the market or not. It is the Fisherian view that enables development of broader measures of asset valuation and prices.

At the heart of the difference between Fisher's and Hick's views is the treatment of expected capital gains under the heading of income. Hill and Hill (2003) make clear that unexpected capital gains are not to be treated as income. Hicks combines expected capital gains into income summing Fisher's "interest-like return" with the expected capital gains. Fisher, on the other hand, simply focuses on the "interest like return" or what might be called net benefit or net rent, and then sums those through time to arrive at net present value, carefully tracking the changes in the underlying stocks.[19]

When attempting to measure changes in the value of assets, i.e., the asset price of stocks, Fisher's view of income is more helpful at an operational level. Sefton and Weale (2006) show that for aggregate wealth across a closed population, capital gains arising from fully owned assets or interest rate changes should not be included in income, despite starting their analysis with Hick's income definition. Ultimately, the capital gains issue is about what to include in income as opposed to how to compute the net present value. Phrased differently, the issues lie in defining the production, asset, and income boundaries of the accounts – something that is in part a political decision about what to include in the social welfare or political preference function and in part practical decision.

Importantly in relation to defining boundaries, Fisher, Hicks, and others do not confine income to money income. This raises a number of practical, but not fundamental, challenges for national accounts (Nordhaus 2006). A key challenge is aggregation of non-money incomes (utilities). A first step is to create money metric demand curve at the individual level. This raises the challenge of defining the "extent of the market," which can have a greater effect on changes in value than estimates of marginal prices (Smith 1993; Adamowicz et al. 2019). Specifically, it is not sufficient to value changes for people who use the resource at time zero, since the users may enter or exit in the future. Furthermore, the time at which individuals intend to start using a resource and receiving income may change between time periods zero and one, which influences the price of a stock, because it impacts the net present value of future uses (Fenichel, Abbott, and Yun 2018).

For any asset held in trust there is the issue of how the income is distributed. This is particularly an issue for natural and ecosystem assets because the trustee may not formally distribute the income, e.g., following Krutilla (1967) a government may be the "trustee" of a national park, and "the existence of a grand scenic wonder or a unique and fragile ecosystem [in that park is] … a significant part of the real income of many individuals." The government does not formally write cheques that are sent in the mail and recorded as bank transactions and yet, the trustee and society actively forgo current opportunities to exchange them with future generations. This is an extreme case when the "owner" of the asset may not be the receiver of the "service." A Coasian bargaining outcome would potentially appear in bank transactions, but what about when that transactions occurs though the negotiation or rights or the political process? Should the presence of transaction costs mean that forgone opportunities to produce a service not be counted when the owner of the asset is unable to capture all the service flows? Where the boundary should be drawn is likely a matter of policy or the information needs of policy, but that must balance ideals, relevance, feasibility, and consistency. Conventions are helpful, but should be revisited from time to time.

[17] A third, intermediate view is Lindahl income, see Weitzman (2016) for a greater explanation.

[18] Hulten (2006) points out that what is capital and what is an intermediate good depends on the accounting period. However, for natural and ecosystem capital, it is clear some asset must persist long enough that calling them intermediate goods seems unreasonable.

[19] An important point is that this amount must be the discount sum of contribution to value, less costs, as shown in Figure 2. Properly normalized transaction values using index number theory or consumer surplus are both reasonable approximations for accounting purposes.

For market goods, readily measured in money terms, individual demand curves are summed over quantities (often called horizontal summation). Valuation of the service or the good is then measured at the market marginal value or observed market price – given the market feature of a law of one price. Further, some goods are "near market" and market prices can reasonably be inferred without too much effort.

For non-market goods imputing prices is a non-trivial task. For these goods, the individual demand curves must (generally) be summed vertically, over marginal value, because there is no law of one price and consumers are forced to consume a common quantity (e.g., air quality) (Freeman 2003). The implicit assumption associated with using a money-metric equivalent for non-market goods is that when individual demand curves, and hence income, that Fisher says "does not flow through the cash draw" is summed to an aggregate income statistic, individuals are weighted by the inverse of their marginal utility of income (Negishi 1960). As the wealthy tend to have lower marginal utilities of income, this process weights the consumption of the rich more greatly than the poor. This is an unavoidable feature from using money as the unit of account without further welfare weighting on income. There are ethical and policy concerns with this weighting scheme. Markets clearly can price discriminate, weighting individual's money differently, based on the individual's observable characteristics (e.g., student and senior discounts). There is a well-known unresolvable challenge in economics associated with interpersonal comparisons (Gilboa 2009; Fleurbaey and Blanchet 2013), but such comparisons are implicit in any measure of national income. It is not our intent to dismiss or marginalize these issues, but rather to note that we simply cannot address them in this paper.

Two additional concerns with the idea of income need to be addressed: production process effects, particularly for non-market services and the idea of negative income. First, consider production process effects. Boyd and Banzhaf (2007) argue that is hard to define a service that singularly flows from ecosystems, and often inputs from produced and human capital are needed to generate a service flow. It is unlikely that such production processes are purely additive. Indeed, ecosystems may enter the production process in multiple ways in a non-additive fashion.[20] This means separating the income solely attributable to the ecosystem or the average ecosystem income may not be a very useful exercise. However, it is possible to measure the marginal income from ecosystems or loss of income from damage, but only incrementally (Muller, Mendelsohn, and Nordhaus 2011). This implies that asset prices for natural or ecosystem assets depend on the broader technological and institutional contexts.

Connected to the production process effect problem is the broader challenge of making income measures comparable over time (beyond the standard index number challenges). This is particularly problematic for income flows that are not obtained through the market and/or are non-rival. This problem is beyond the scope of this discussion paper, but there are likely strong parallels with pricing challenges associated with intangible (e.g., digital) assets and their associated income flows (Nakamura 2010; Brynjolfsson, Collis, and Eggers 2019).

A feature of any real asset is that it may contribute to income, defined as an interest like return, in multiple ways simultaneously, i.e. providing a bundle of different services. For example, a house may provide protection from the elements and provide leisure opportunities. For natural and ecosystem assets, the breakdown of the idea of one asset-one service is common since they often provide multiple services.

Next, consider the idea of negative income. There is nothing in the Fisher or Hicks concepts of income that guarantees their respective income concepts will remain positive. For example, in the case of Hicksian income, it is possible that in certain instances no amount of current savings will enable equivalent future consumption, especially if the underlying capital is rapidly and exogenously deteriorating. In such a case, Hick's income concept may require payments in the present to avoid greater payments in the future. Likewise, Fisher's interest like return may not be positive, net of a sort of "servicing cost." Moreover, this may occur while assets are deteriorating, leading to an expected capital loss. Consider the case when adding more to a stock, e.g., CO2 in the atmosphere, leads to a decline in the net present value of income. Because the asset price is defined as the marginal change in

[20] Nevertheless, linear input-output (IO) tables are important for policy analysis, so approximations need to be made.

net present value with respect to a change in the quantity of the stock, the price must be negative. In cases where a stock is so plentiful that society would exchange other resources to be rid of the stock, e.g., the stock of HIV virus or perhaps water in coastal areas prone to flooding, rather than being an asset, the stock might be considered a liability and have a negative price. One interpretation is that current users are acting as if future users should compensate the current users for producing the stock that is eroding future consumption opportunities. This statement is not as absurd as it first appears. From an accounting perspective however, an initial challenge is to ensure appropriate alignment of relationships among stocks, service flows, asset owners and beneficiaries and definition of measurement boundaries such that the measurement of asset prices is appropriately targeted.

3.5 Depreciation and regeneration

In relation to produced assets, Jorgenson (1963) emphasizes the importance of net depreciation or deterioration in valuing assets. Jorgenson thinks of this as the "replacement rate" necessary to maintain a constant stock. But, if the stock is declining and replacement does not occur, then the Jorgenson measure is a depreciation or deterioration rate. This deterioration rate adjusts for the *opportunity cost of holding capital*. In a simple setting, the discount rate is the opportunity cost of holding capital. If the net deterioration rate is positive (i.e. the asset is depreciating), this increases the opportunity cost of holding capital. On the other hand, if the deterioration rate is negative, e.g., the asset is increasing in quantity, than the asset is appreciating. This then acts to reduce the discount rate because the opportunity cost of holding a growing stock is lower than holding a constant stock.[21] Natural resource economists have usually just called this a "stock effect" (Clark 2005).

Generally, produced assets are assumed to decline in value over time and commonly a focus is on establishing an estimated asset life and depreciation profile. On the other hand, many natural and ecosystem assets are able to regenerate naturally and hence can physically appreciate or depreciate. For example, a change in the fish stock may enable greater total reproduction leading to faster appreciation but also attract more predators (or fishers) leading to faster depreciation. Which effect dominates will be system specific.

Horan et al. (2018) show that balancing a portfolio of natural assets with varying rates of appreciation and depreciation is challenging even without uncertainty. These challenges are conceptually similar to economic general equilibrium concerns related to time varying interest rates. We focus on the *net* depreciation because multiple forces can act to change stocks at the same time.

From a national accounts perspective, the focus on net depreciation is appropriate in broad terms but will be reflected in a more extensive set of accounting entries involving recording changes in stock between opening and closing stocks (examples include investments, discoveries, extraction, catastrophic loss). These entries will, in nominal terms, fully account for the change in nominal values of assets over time. Additionally, it is necessary to consider the expected future patterns of change and this is where the focus on net depreciation comes into play. In national accounts, expectations of change are often based on past patterns of changes as recorded in the accounts. There is a potential bridge between these past patterns, the economic ideas of the resource allocation mechanism or economic program and the ideas just described around net depreciation and opportunity costs.

Abbott, Fenichel, and Yun (2018) building on prior work, considered future changes and the roles of deterministic and stochastic physical changes in stocks. They show that the deterministic or expected changes in stocks, i.e., the "stock effect", add to or subtract from the opportunity cost of holding capital, but may also impact the expected capital gains of other stocks (Yun et al. 2017). Moreover, Abbott et al. find that the variance terms influence capital gains of own and other real assets. Understanding net depreciation is therefore particularly important for natural or ecosystem assets, and this is where science explicitly enters the valuation process. Natural resource

[21] Monetary inflation is basically hold a constant physical stock of capital while demand for capital investment rises, which causes the money supply to contract raising prices. This is a general equilibrium effect. The partial equilibrium equivalent to hold the value of money constant and reduce the capital stock.

management has long held that many natural assets can be managed in perpetuity with zero net depreciation over time, e.g., regenerating forests and fish stocks. However, broader pressures, e.g., climate change and urbanization, may be changing this mindset. Furthermore, ecologists are increasingly questioning the steady state model of ecosystems (e.g., Hastings 2001) just as economists have done for macroeconomic dynamics (Nordhaus and Tobin 1972).

3.6 Discount rates

A discount rate is needed for the purposes of valuing or pricing natural or ecosystem assets. Many national authorities establish such rates, which are commonly referred to as social discount rates where the word social refers to the idea of "aggregate" or to be applied to something of material importance to all of society. However, as Baumol (1968) writes, "few topics in [economics] rival the rate of social discount as a subject exhibiting simultaneously a very considerable degree of knowledge and a very substantial level of ignorance."

The SEEA Central Framework (UN et al 2014) provides a summary of discount rates from a national accounting perspective and Fenichel, Abbott, and Yun (2018) propose that national rates should be used to facilitate "apples to apples" comparisons, but this a practical rather than rigorous argument.

Asset pricing for financial assets has largely focused on the private term structure for the asset (Fama and French 2004; Perold 2004). This is often accomplished by measuring risk relative to an asset with a known and risk-free term structure – which is often assumed to be a constant rate of return. This constant rate of return or opportunity cost of capital is also interpretable as a private discount rate. A constant rate of return that is an exponential process leads to an exponential discount rate.

Exponential discounting has a number of desirable properties that make it a good starting point for the discounting process that is required for measuring the price or change in value of assets not traded in the market.[22]

- First, exponential discounting is time consistent, which means that an agent who discounts exponentially does not regret past decisions solely because of the passage of time (Dasgupta, Maler, and Barrett 1999). This is important in the context of utility or welfare, but it is less clear that this feature is required of money. Indeed, discounting processes that are time consistent in welfare are often not time consistent in money. Millner and Heal (2018) discuss the time consistent property and relate to another desirable property of time invariance.
- Second, exponential discounting leads to finite net present value over an infinite time horizon.[23] This enables the analyst to avoid choosing arbitrary planning horizons. Fleurbaey and Blanchet (2013) suggest the choice of discount rate is really about how much "future warning" one wants. This implies that even if the accountant wished to fully depreciate an asset in a short time period, then the account could simply choose a greater discount rate. However, the exponential process is a practical choice when the asset could potentially persist in perpetuity.
- Third, most public accounting authorities use exponential discount rates, despite theoretical evidence that perhaps social discount rates should be time varying (Arrow et al. 2014). Even governments that have adapted declining discount rates for the social discount rate often do so in long steps, making the

[22] Other discounting processes, that lead to time varying discount rates have attracted substantial attention (e.g., Arrow et al. 2014; Gollier and Zeckhauser 2005; Goulder and Williams 2012; Weitzman 2001; Weitzman 1998; Freeman and Groom 2014). The topic probably merits a separate paper. In the interest of tractability and for practical reasons, we focus on exponential discounting.

[23] Indeed, all thin-tailed distributions or discounting processes are guaranteed to integrate to a finite value, and some fat-tailed distributions can also integrate to a finite value, especially with bounds on support (Pindyck 2011). Jamison and Jamison (2011) show that the different discounting processes can lead to the same net present value but with different speeds of convergence.

discount rate effectively exponential. For example, the United Kingdom uses a declining social discount rate, but holds the rate constant at 3.5 percent for the first 30 years and uses an exponential discounting process (Lowe 2008), which leads to 60 percent of the value discounted away through exponential discounting by the end of the first step.

Overall, the use of exponential discount rates, which is consistent with the advice in the SEEA Central Framework, is likely a reasonable starting place for the purpose of national accounting for natural or ecosystem assets.[24]

For private discount rates, which may vary through time, money is a reasonable object of measure – particularly for the investor. The individual is left to assign for him or herself the value of that money itself through time. The theory of social discount rates is more complex, given the potential for a time varying value of money (determined in general equilibrium); the theory of social discounting starts with a utility or welfare discount rate (Dasgupta, Maler, and Barrett 1999).

3.7 The allocation mechanism

The value of an asset is the expected net present value of the service flows it provides. Many economists have focused on the change in value of assets or capital assuming an optimal allocation of assets (Asheim 2000; Hamilton and Ruta 2009). However, the use of this assumption is generally to provide a baseline valuation in a world with no externalities. The broader purpose of most this work is to show how the accounting should work and to connect to well-developed theories in welfare economics. Nordhaus and Tobin (1972) make this clear in the distinction among actual (the object of accounting), sustainable, and potential measures.

Arrow, Dasgupta, and Maler (2003), Dasgupta and Maler (2000), and Dasgupta (2007) argue that the actual valuation must reflect real institutional contexts. Nordhaus (2006) states, “we clearly need to recognize that nonmarket assets have value, and that their value can be increased or decreased through human activities,” which implies that their marginal value or price cannot be assessed without considered the expected human activities. Indeed, if a society locks in an inefficient and suboptimal allocation mechanism, then it can hardly expect to realize the value associated with the optimal allocation (Fenichel and Hashida 2019). Thus, assuming an optimal allocation mechanism is unwise and it is also unnecessary for valuation for accounting purposes (Fenichel, Abbott, and Yun 2018). Though, it may be of interest and importance in policy analysis.

Importantly from a SEEA perspective, assuming an optimal allocation of assets is inconsistent with the goal national accounts to measure actual income and production. In justifying using prices based on observed behaviour to value air pollution, Muller, Mendelsohn, and Nordhaus (2011) write, “The economy has many existing distortions other than those from air pollution—such as taxes, distortions from market power, and other externalities—and existing accounts do not attempt to incorporate those.” Therefore, a particular challenge to valuing natural and ecosystem assets is the assessment of the actual allocation mechanism that is in place and what is expected in the future. This is more challenging than suggested in Muller, Mendelsohn, and Nordhaus (2011), because those authors only measure a static contribution to production. Fenichel, Abbott, and Yun (2018) approach the “expected in the future” part of the question by modelling the allocation mechanism as a feedback process of the state of world and asserting that, barring alternative evidence, the current feedback process best describes the future feedback process.

The consideration of natural and ecosystem assets introduces challenges into a complex theory of income, welfare, and prices. First, ownership of many aspects of these assets is seldom clearly defined, except perhaps in a somewhat vague notion of a public trust. Yet, clear and enforceable ownership is a necessary condition in Sefton and Weale’s (2006) treatment and for the asset boundary in the SNA (European Commission et al. 2009), especially when services are provided apparently in-kind because of institutional arrangements. This complicates the already difficult problem of aggregating heterogeneous people with different preferences whose income may be enhanced

[24] Methods developed in Fenichel, Abbott, and Yun (2018) could be used to address step-wise declining discount rates.

by natural and ecosystem assets but do not exchanges in the market for the service that capital provides. Second, most of the theory on income and welfare measures requires an efficient or optimal economy – an unmaintainable assumption in the context of natural and ecosystem assets and one not actually reflected in the national accounts. Third, theories of income and welfare require information on preferences. In practice, national accounts estimates provide approximations for changes in welfare, as suggested in the first section, based only on observed prices and quantities – hence the motivation for and use of index number theory (Diewert 1992; Fleurbaey and Blanchet 2013). These prices and quantities incorporate revealed preferences. Yet for natural and ecosystem assets, prices often need to be imputed, which begs the question of how far it is reasonable to strictly adhere to an approach based on price-quantity measures.

With these challenges in mind, we move to define prices for natural and ecosystem assets that are directly comparable to observed market prices and that are useable in price-quantity based measures of change. These prices are defined based on observed and forecasted behaviour. We also discuss the extent to which using observed behaviour to impute prices may provide more precise approximations of welfare changes than common price-quantity measures. These approaches may be especially important given the market failures associated with many forms of natural and ecosystem assets.

4. Ecosystems and natural resources as assets

A concept of total asset value, in the sense of $P \times Q$ - especially with regard to the environment, is not useful.[25] Nevertheless, it may be credible to ask how the value of substantial components of the environment change through time, especially when the changes themselves are relatively small. The value of these changes may be the more important macro indicator for ecological wealth management and for understanding the potential for future human wellbeing. The important point is that there are changes in natural and ecosystem assets that can be plausibly valued using currently available data. The valuation of doomsday scenarios is beyond the scope of credible information, but then we hardly need accounts to tell us that we should avoid the end of the world.

Fortunately, applying asset valuation theory to nature is not new, and there is "no need for a new and grandiose theory of natural resources," since natural and ecosystem assets are a critical part of society's capital and should be treated as such (Scott 1973). Indeed, the first example of a capital stock in Irving Fisher's 1906 text was the Newfoundland fish stock, and non-produced fish stocks are within the asset boundary of the SNA (European Commission et al. 2009). Economists have long conceptualized natural resources as assets (Gaffney 2008), and the extension to ecosystems (Barbier 2011, 2013) simply puts ecosystems in the context of biophysical feedbacks that influence changes to the assets beyond human decisions.

However, for a price or change in value to exist, it is important to be clear about exactly what is changing. For example, in the context of the environment, people may talk about the value of water. What must be considered is not simply the value of the change in water, but specifically whether the change concerns the quantity of the water or some attribute of its quality, e.g., dissolved oxygen or some other variable.

To start discussion, consider that natural assets are stocks of specific biological or physical attributes that are produced through natural processes and contribute to generation of a service to people in the present or in the future. The hallmarks of natural assets are

- that they are produced by nature in ways that do not respond to market signals;
- that they have a single clearly defined unit often associated with a geographic region;
- that their allocation mechanisms can be simple or complex; and

[25] Perhaps a more useful notion of "total value" is the expected net present value of holding a given quantity of an asset, which may not change linearly with the quantity of asset.

- that they embody some opportunity to provide a value added service directly or indirectly to people.

Examples include, cubic meters of groundwater in Kansas, kilograms of cod in the North Sea, tonnes of coal in Inner Mongolia, and the number of woodland caribou in Alberta.

Not all natural assets are part of an ecosystem (e.g. coal deposits). By contrast, ecosystem assets are always spatially defined, comprise many different stocks that could each be considered natural assets, and are often managed through multi-faceted allocation mechanisms. Given that natural assets may be nested within ecosystem assets, we first discuss pricing natural assets.

4.1 Natural assets

The idea that natural resources are assets is old, but it has been a challenge to operationalize in an accounting context (Polasky et al. 2015). Nevertheless, the asset price for a natural resource can be computed using the method developed in Fenichel and Abbott (2014) and extended and refined in Fenichel, Abbott, and Yun (2018). The general approach follows Jorgenson (1963), and states that the price of a real asset, including a natural asset, is the change in real (Fisherian) income with respect to a change in the stock plus expected capital gains (explained in detail below – these forward looking capital gains are associated with a non-constant expected dividends flow) all divided by the discount rate adjusted for net depreciation (opportunity cost of holding capital). Hulten (2006) argues that this is the proper pricing concept for assets in national capital accounts.

The Fenichel and Abbott approach uses the fact that expected (*ex ante)* capital gains can be described as a derivative of the asset price with respect to time.[26] Therefore, the price and capital gains terms can jointly be approximated using functional approximation techniques based on estimates of (i) the marginal effect of a change in stock on net depreciation, (ii) the marginal effect of a change in stock on real income, and (iii) a discount rate (Fenichel et al. 2016; Fenichel, Abbott, and Yun 2018). Furthermore, real income flows and changes in stocks may also be used to jointly approximate the net present value flow expected from the asset and the asset's price (Fenichel, Abbott, and Yun 2018).[27] Yun, Fenichel, and Abbott (2017) provide an R package for doing the numerical approximation, which we demonstrate in an example in the following sub-section. Here we explain the valuation and approximation concept for the interested reader, however these next three paragraphs may be skipped by those not interested in these details. We present the approximation theory for the single stock deterministic case and refer the reader to the Yun et al. (2017) for an extension to multiple interacting stocks or to Abbott, Fenichel, and Yun (2018) for cases with stochastic dynamics.

> Consider a single stock of natural capital, $s(t)$, at time t that may take value over a finite interval.[28] The change in that stock through time, $ds/dt = \dot{s}$, is driven by environmental or natural process $G(s(t))$ and human impacts, which may be written a feedback rule (conditioned on institutions) $F(s)$. Thus $\dot{s} = G(s) - F(s)$, where we follow convention and assume human impacts adversely impact stock growth, e.g., harvesting, but note that $F(s) < 0$ if people actively increase stock s. While we use harvesting as the example, $F(s)$ may also include incidental impacts. People also gain a net benefit or "interest like

[26] The Fenichel and Abbott approach assumes that the passage of time has no pure effects on income. Arrow, Dasgupta, and Maler (2003) suggest this is not a serious constraint because additional stocks can be added to the system. Asheim (2010) suggests some generalizations to the Arrow, Dasgupta, and Maler framework for addressing pure time effects. In theory, the Asheim or Arrow et al. approaches appear sound, applying them in practice has some challenges.

[27] Formally, Fenichel, Abbott, and Yun approximate the integral under the multi-dimensional price surface, which is the net present value. Differences in the net present value can be used to find changes in value associated with exogenous institutional shifts, but care needs to be taken not to extrapolate too far out of the data.

[28] We drop the time notation of (t) unless doing so causes confusion. Abbott, Fenichel, and Yun (2018) and Fenichel, Abbott, and Yun (2018) provide extensive details for extending the logic shown here to multiple interacting stocks and stocks with stochastic dynamics. The purpose here is to formalize the concept.

payment", W, from s at each time t that depends on the quantity or condition of $s(t)$; this is written $W(s(t))$, which is net of costs. In the case of an extractive industry, this is resource rent. Assume that s may, in principle, persist forever (or at least a very long time), but that society discounts future flows of W using an exponential process at rate δ. The net present value of s is, $V(s(t)) = \int_t^\infty W(s(\tau))e^{-\delta(\tau-t)}d\tau$. $V(s)$ is the *value* of asset s at a given point in time – this is the only sense in which a "total value" can exist. This of course, depends on how s is forecasted to change, using $\dot{s}$, over the interval t to infinity. Without relying on optimization, Fenichel and Abbott (2014) show that:

$$[1] \qquad \delta V(s(t)) = W(s(t)) + \dot{s}(t)V_s(s(t))$$

which Shapiro and Stiglitz (1984) call the fundamental asset equation, Weitzman (1976) shows this is also an expression for net national product if there is only a single stock of capital, and the expression is also known as the current value Hamiltonian.[29]

The expression implies that $V(s(t))$ can be rewritten on as $\delta^{-1}\Big(W(s(t)) + \dot{s}(t)V_s(s(t))\Big)$. Furthermore, recall that $\partial V(s)/\partial s = V_s = p(s)$, is the price of asset s, but that this price may vary over time with changes in the quantity of s. What determines whether this is a market price or revealed shadow price is whether or not it is directly observed, but the relationship holds for market assets too. $p(s)$ is price P_1 at quantity $s_1 = Q_1$, that is point A in figures 1 and 2. Therefore, since price is only $V_s(s)$, the price alone does not imply value $V(s)$, or more precisely the value of *exchanging* the stock from a level s_1 to s_2, is $V(s_1) - V(s_2)$. The change in value may be found by integrating under the price curve or establishing a constant accounting price, $\bar{P}$ using index number theory (see figure 2), and multiplying $\bar{P} \times (s_1 - s_2)$.

A total value of stock $V(s_1)$ is found the same way by setting $s_2 = 0$. If the stock price of s is infinite (e.g., all water), then total value is nonsensical since the index number theory based accounting price is also infinity, e.g., the Fisher Ideal price index produces $\sqrt{P_1 \times \infty} = \infty$.

Using subscripts for partial derivatives, $p(s)$ can be written as

$$[2] \qquad p(s) = \frac{W_s(s) + \dot{p}}{\delta - (G_s(s) - F_s(s))}$$

In the case where time only enters through the dynamics of s, the capital gains term, $\dot{p}$, can be written as $\dot{p} = p_s\dot{s} = V_{ss}\dot{s}$. This is algebraic rearrangement of Jorgenson's (1963) formula for the price of invested capital with taxes set to zero. Examples, of how $p(s)$ is measured in practice are presented in Fenichel and Abbott (2014) and Fenichel et al. (2016), which can be calculated based on equation [1] or [2] depending on the information available. Fenichel, Abbott, and Yun (2018) explain that using equation [1] has computational advantages, especially when dealing with multiple stocks, but that most estimates only measure derivatives so using equation [2] in some cases might introduce less approximation error. Either way, the general approach to approximation relies on standard functional approximation techniques.

Fenichel, Abbott, and Yun (2018) (FAY) describe three different approximation techniques and explain the pluses and minus of each. All will give the same answer with sufficiently rich data and sufficiently continuous processes. Here we show what FAY call "value function approximation" because it has the simplest notation to explain the core idea. FAY argue that if dynamics do not depend explicitly on time, then the current value Hamiltonian can be written down of a N-length vector values of stock s over finite interval expressed as $\boldsymbol{s} = [s_1, s_2, s_3 \ldots s_N]^t$. The function $V(s)$ and $V_s(s)$ are unknown. However, $V(s)$ can be approximated with a polynomial that increases in precision with the order of the polynomial (Judd 1998). The polynomial is comprised of basis functions $\boldsymbol{\mu}(s_i)$ for each element i in the vector $\boldsymbol{s}$ and a

[29] In other contexts, Stokey (2008) and others show that the derivation of the current value Hamiltonian does not depend on optimization.

vector of unknown parameters β. The numerical value of basis functions are determined only by the domain of approximation and the order of the approximation. FAY use Chebyshev polynomials basis functions, which have desirable computational properties (Judd 1998; Vlassenbroeck and Van Dooren 1988; Press et al. 2007). There is an association set of basis functions $\boldsymbol{\mu}_s(s_i)$ that when combined with β provide an approximation of the derivate V_s. Therefore, an N-length vector of approximate versions of the current value Hamiltonian relationship can be written as $\delta\boldsymbol{\mu}(\boldsymbol{s})\boldsymbol{\beta} = W(\boldsymbol{s}) + diag(\dot{\boldsymbol{s}})\boldsymbol{\mu}_s(\boldsymbol{s})\boldsymbol{\beta}$**,** where function are applied element by element to $\boldsymbol{s}$, the vector of known parameters, $\boldsymbol{\beta}$**,** has length $k \leq N$ and diag refers to square matrix with the vector $\dot{\boldsymbol{s}}$ on the diagonal and zeros elsewhere. The solution for the vector $\boldsymbol{\beta}$ is the generalized inverse,

$$\boldsymbol{\beta} = \left(\delta\boldsymbol{\mu}(\boldsymbol{s}) - diag(\dot{\boldsymbol{s}})\boldsymbol{\mu}_s(\boldsymbol{s})\right)'\left(\delta\boldsymbol{\mu}(\boldsymbol{s}) - diag(\dot{\boldsymbol{s}})\boldsymbol{\mu}_s(\boldsymbol{s})\right)^{-1} \times \left(\left(\delta\boldsymbol{\mu}(\boldsymbol{s}) - diag(\dot{\boldsymbol{s}})\boldsymbol{\mu}_s(\boldsymbol{s})\right)' W(\boldsymbol{s})\right).$$

The vector $\boldsymbol{\beta}$ can be used with the function $\boldsymbol{\mu}(s)$ to get a measure for net present value that can be used to compute non-marginal changes in value or with $\boldsymbol{\mu}_s(s)$ to get prices. The greater values of N on a given interval and small values of $N - k$ will lead to increasing precision of the approximation.[30] Furthermore, a basis $\boldsymbol{\mu}_{ss}(s_i)$ can be found to explore capital gains effects. The approximation approach can be extended to multiple stocks (Yun et al. 2017), stochastic dynamics (Abbott, Fenichel, and Yun 2018), and with non-convexity and discrete actions (Hashida and Fenichel 2019).

Capital gains, defined here as the components of an asset's price that come from expected changes in the quantity and/or price of the asset or other assets, are not a primary interest, but are an implicit part of the approximation process, and the capital gains terms may also provide useful information about the system. All capital gains discussed are *ex ante* and are based on creditable forecasts rather than *ex post*, which is important for capital gains to be included in valuation (Hill and Hill 2003). Abbott, Fenichel, and Yun (2018) decompose the capital gains term into seven components.

- The first component is a deterministic own-price effect. The effect relates to how changes in the stock itself are forecasted to effect price – in essence a scarcity effect. Gollier (2019) points out this effect may offset the effect of discounting thus leading the present value of a stock to rise over time.
- The second component is a deterministic cross-price effect. This is an effect that relates to substitution and complementarity with other stocks (and the Slutsky substitution matrix). It reflects how changes in stocks of other capitals may impact the marginal value of the stock of interest.
- The third component is a deterministic cross-stock effect. This reflects how changes in another stock may impact the physical state of the stock. These latter effects are especially important in the context of ecosystems, because they capture equilibrium processes happening outside of the economic system – such general equilibrium effects are important in a system of national accounts.
- The remaining four components relate to risk. These can be summarized as two kinds of capital gains effects one working through own-stock risk (variance terms) and one working through cross-stock risks (covariance terms). The first of the stochastic terms is an endogenous risk effect. It is associated with the idea of a risk premium, and generally lowers the value of an asset if the future of that asset is more volatile. This the risk effect that Gollier (2019) focuses on. The second stochastic term is an endogenous risk aversion effect. The dynamics, including the volatility of dynamics impact the curvature of the value function – the function describing the net present value of services flows as a function of the stock of the asset. This effect is similar to the prudence effect in precautionary savings literature (Kimball 1990). The potential richness of these effects on the expected capital gains may be informative about the processes ultimately influencing natural (or ecosystem) asset prices.

In order to impute natural asset prices, empirical evidence is need about the dynamic processes affecting the physical availability of stocks in the future. This is a separate challenge from assessing the value of the service flows associated with those stocks in the future. Equation [2] is a generalization of the net present value of annuity that is commonly taught in finance classes, in which case the second numerator and denominator terms are set to

[30] See Fenichel, Abbott, and Yun (2018) for more details, sources of identifications, and potential numerical complications.

zero. Based on application of the approximation approaches described above, ignoring the $\dot{p}$ term in Equation [2] might not be horrible, but is generally unnecessary. However, failure to include all the denominator terms in equation [2], which is sometimes used along with the claim that it is the net present value of rents, can lead to price curves that slope up when they should slope down (suggesting gains rather than losses from decreases in stocks) and lead to orders of magnitude difference in the asset price (see figure 4 in Fenichel and Abbott 2014). The reason for this is that marginal dividends are non-constant and expected to change over time. Forecasting the future availability of stocks requires measuring non-human (natural) and human drivers for stock change. In the case of a single stock natural asset valuation, other ecosystem inputs are held fixed and treated as parameters. For example, in modelling fish dynamics, nitrogen concentration in the water may be treated as fixed.

Ecosystem capacity considerations may enter at this stage. Capacity may be better thought of as conditional capacity or capacity for the stock to exist or grow at a certain rate rather than the overall maximum "carrying capacity" of the ecosystem for the stock. In some cases, and under some assumptions, these may be the same thing. Capacity and the idea of local or marginal capacity takes on increasing importance in the context of ecosystem assets where component natural assets may all be treated as changing through time.

There is increasing recognition that people are a driving force on earth environmental systems, perhaps to the extent of ushering in a new geologic epoch – the Anthropocene (Lewis and Maslin 2015). This means that biophysical understanding of the determinants of changes in natural asset stocks is not sufficient and understanding human impacts on natural and ecosystem assets is important. Fenichel, Abbott, and Yun (2018) suggest that if the feedback processes are fast, then they can create a behavioural equilibrium. They go on to argue that the feedbacks can be made arbitrarily fast by increasing the dependencies on stocks of other assets. Then, if (1) feedback rules are solely a function of stocks and (2) economic program or resource allocation mechanism is time autonomous, then the dynamics are fully captured in the natural stock dynamics. This can include stochastic processes. Fenichel and Hashida (2019) discuss the importance of modelling the allocation mechanism.

The importance of credible future forecasts makes measuring prices for natural assets challenging. The key empirical challenge is to identify how changes in stocks of assets impact changes in the income flows and changes in the dynamics of the stock under the expected set of institutions – which barring contrary evidence is likely the current set of institutions. It is important to recognise that this is different from a prevailing concern in economics on the identification of parameters that enable statements about income or system dynamics under counterfactual institutional arrangements, which is important for benefit-cost analysis. Furthermore, in the example that follows we show that assumptions about the process of institutional change that leads to observed price-quantity pairs can have large effects on estimates of changes in value.

4.2 An example of natural asset pricing

Fenichel and Abbott (2014) use existing ecological and social data to measure the accounting asset price function for Gulf of Mexico reef fish in US waters.[31] Fenichel and Abbott's analysis is built on preceding empirical work of Zhang (2011) and Zhang and Smith (2011). Zhang and Smith (2011) used over a decade of highly detailed commercial fishing trip data to estimate parameters of a logistic growth function (r and k) for an aggregated Gulf of Mexico Reef fish stock. The logistic growth equation is used to partially parameterize $\dot{s}$ above in the form $\dot{s}(t) = rs(t)(1 - s(t)k^{-1}) - F\left(x\left(s(t)\right)\right)$. These parameters can be thought of as describing part of the "capacity" of the fishery. Zhang and Smith also estimate the catchability (q) of the fish stock, which is also a "capacity" parameter

[31] Yun, Fenichel, and Abbott (2017) include the parameterization and steps in their analysis, along with some extensions. The fully worked example is available in the capn package for R, https://cran.r-project.org/web/packages/capn/index.html. The example is called by loading the capn package and running the command demo("GOM"). Code for this example is available upon request from the authors.

that summarizes the interaction of the natural capital with produced and human capital.[32] The q parameter is used to partially parameterize the function F, which is part of $\dot{s}$ and addressed in greater detail in the next paragraph.

Zhang (2011) uses the same data set (up to some data cleaning differences) to estimate the behavioural response of fishers to changing levels of stocks (in the capn demo these are α, γ, y), which are used to parameterize the fishing production function. The fishing production function returns how many fish are harvested at time t, which is $F(s(t)) = qs(t)x(s(t))^{\alpha}$, with $x(s) = ys^{\gamma}$. This provides the economic program or resource allocation mechanism. Combining F with the logistic growth function provide the $\dot{s}$ equation and the information necessary to take its derivative. The function $x(s)$ will also enter into the resource rent calculation that informs the function $W(s(t))$.

Fishing log book data and market data are used to compute average prices per pound of fish landed and price per unit effort, x, so that resource rent at time t is $W(s(t)) = price \times \; qs(t)x(s(t))^{\alpha} - cost \times x(s(t))$. This expression along with the logistic growth expression can be differentiated with respect to s to get the marginal dividends function. The logistic growth function can also be modified to include catch so that change in stock with respect to time is $\dot{s} = rs(1 - sk^{-1}) - \; qs(t)x(s(t))^{\alpha}$, which can be differentiated with respect to s to get the marginal net depreciation. The capn package can be used to implement approximation technique to measure the asset price curve (Figure 4, left panel, blue curve). Fenichel and Abbott observe that the fish stock had been around 24 percent of carrying capacity (blue dot), with a price of \$3.08 per pound (solid blue dot in the left panel).

From this basic framing of the stocks and asset prices, consider an institutional change that would keep the parameters of the system the same, but move the stock to maximum sustained yield (50 percent of carrying capacity, indicated by the closed black dot). This is likely a non-marginal change in the stock. Fenichel and Abbott point out that this would have to be a transient shock, because the dynamics of the system, which would be expected to return to the steady state of 24 percent of carrying capacity.

Nevertheless, it is useful to understand changes in value. Integrating under the curve implies an increase in value of \$247 million. Using index number theory in the form $0.5(P_2 + P_1)(P_2 - P_1)$, implies a change in value of \$253 million, a 2.5 percent error. Applying the Fisher Ideal price index suggests to a change in value of \$251 million an error of 1.6 percent. However, computing $P_2 Q_2 - P_1 Q_1 = \$154$ million. This is only 62 percent of the actual change in value, implying that society should be willing to invest substantially less in achieving such a change. Equally concerning, if society were to move from operating at 50 percent of carrying capacity to 24 percent of carrying capacity, then under the *common misinterpretation* of wealth accounting practice of computing change in wealth as $P_2 Q_2 - P_1 Q_1$, society would only need reinvest 62 percent of the true value lost to maintain comprehensive or inclusive wealth.

In order for policy change to make maximum sustained yield the new long-run steady state, policy would need to change a parameter. Whether this policy changes is exogenous or endogenous has substantial repercussions for thinking about changes in value. Assume that policy could be put in place, *exogenously,* that reduced α by 10.3 percent, which Fenichel and Abbott interpret as more strict gear or technical restrictions, or to reduce γ by 9 percent, which Fenichel and Abbott interpret as shift in the behaviour or preferences of fishers. Either shift would lead to a long-run equilibrium at the maximum sustained yield stock size, but these changes have very different economic implications. In the language of Figure 3, point A is the solid blue point and point D is the black open point on the black curve in the case in the shift in α and black open square in the case of a shift in γ.

In the case with no parameter changes, we integrated under the price curves from zero to the two stock levels and took the difference. This meant that all the area to the left of the initial stock size (24 percent of carrying capacity) differenced out. This is no longer the case, because the entire price curve has shifted exogenously. The true price curve no longer passes through the solid blue dot, and the changes in net present value for every stock size are shown in the right-hand panel of Figure 4. The shift in γ greatly increases the value of fishery while the shift in α greatly decreases it. For example, the shift associate with change in α from the blue solid dot to the black open dot

[32] Zhang and Smith actually estimate many catchability parameters that vary by gear type and location. Fenichel and Abbott use an aggregation of these parameters in their analysis.

leads to a net present value change of -$435 million. This difference reflects the loss in net present value of future earnings despite the fact that stock has increased. Decreasing α means fishers must use more effort to catch fewer fish, greatly reducing resource rents.

What if only the blue solid dot to the black open dot were observed, which no knowledge about the mechanism for the shift? In this case, we might assume that the institutional change was endogenous. Therefore, the true unobserved price curve would pass through both points. In the case, we would need to use index number theory. In the case, the index number theory calculation must lead to an increase in value, because quantity increases and prices are positive. Index number theory assumes that the price-quantity shifts are endogenously part of a general equilibrium system so that parameters cannot shift exogenously. The implicit assumption is that if the stock declines back to 24 percent of carrying capacity after being at the maximum sustained yield level, then the system will revert to the old institutional arrangements. Applying index number theory to calculate the value changes seems consistent with national accounting, but also missing an important feature of this hypothetical example. Applying the Fisher Ideal index suggests that the shift from the solid blue dot to the open black dot increases value by $206 million. However, the difference in the net present value integrals is (seen in the right-hand side figure).

This comparison illustrates why the institutional arrangements and the process of institutional change matter for non-market asset valuation. Unfortunately, in the real world it may not be obvious whether the observed shift was in endogenous or exogenous. This raises the question what other information is permissible in a system of accounts beyond prices and quantities? In practice, if the behavioural function were estimated with data from a time series associated with the shift, then we would likely estimate an average α, implicitly treating the shift as endogenous, for example a constant institution but where α itself has a feedback rule as a function of s, e.g., $\alpha = \alpha(s)$. Alternatively, we could create an "institutional stock" of α, which would reveal the lost value. In this case, there might be a large increase in the value of the natural capital, but a large decline in the value of institutional capital. In net, the two would lead to loss of $435 million.

In taking forward the SEEA EEA revision process it is intended to work through additional examples of the application of the valuation theory and approach described in this paper, including describing the connections (e.g. concerning methods, assumptions, data sources) to the way in which NPV calculations are commonly undertaken by national accounts compilers for the purposes of SNA balance sheet estimates of natural resources.

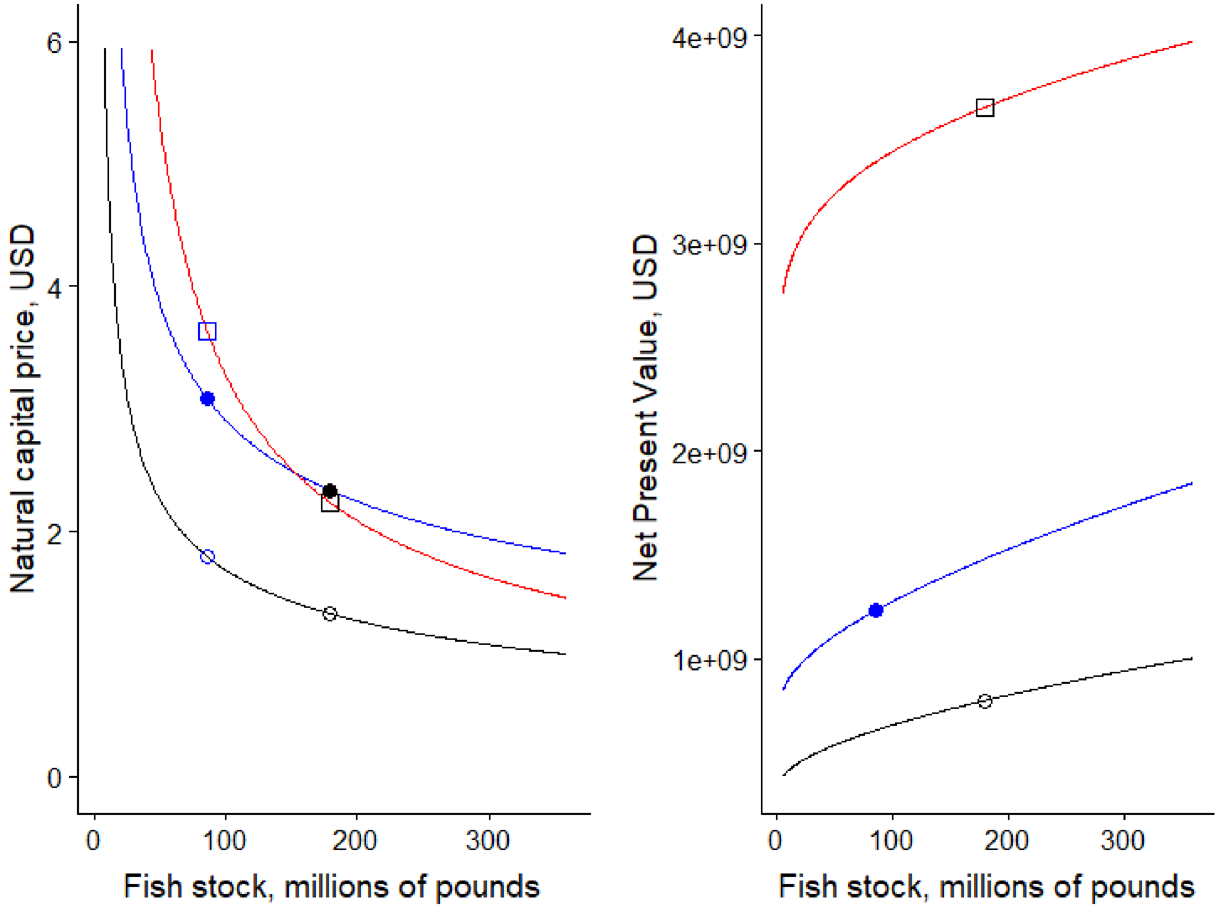


Figure 4. The asset price curve (left) and value function (right) for the Gulf of Mexico example. The blue curves are the empirical specification. The black and red curves represent shifts in behavioural parameters α and γ to achieve maximum sustained yield as long-run equilibrium. In the net present value plot only long-run equilibria are shown.

4.3 Ecosystem assets

The SEEA-EEA focuses on ecosystem assets, but the revision of the SEEA-EEA is also an opportunity to clarify the connection between ecosystem assets and natural assets. The key distinction between natural assets and ecosystem assets is the *system* component. The reason to value an ecosystem asset is to recognize the value of the ecological processes, under a specific resource allocation mechanism. The resource allocation mechanism describes how people are impacting and benefiting from those ecological processes.

Natural assets are defined by their precise units. However, these units can be challenging to determine because natural assets are difficult to arbitrage and spatial location can matter intensely (Addicott and Fenichel 2019). Furthermore, the attributes of the stocks of natural assets can vary. For example, one may define wildlife as a natural asset. Within that asset there are different species, within species, age, sex, size and other features may matter in addition to location. This is similar to Nakamura (2010) concerns about asset valuation in an innovation economy. However, unlike Nakamura's case, all of these subdivided stocks may interact to create a *system*, and the composition of the system may affect the marginal value of any one stock. This reality is all the more reason for an ecosystem asset approach. Of course, one challenge is that stocks of produced and human capital may change as part of the larger system (see prior example). These influences need to be tracked and accounted for, but SEEA-EEA currently excludes these assets from the scope of ecosystem assets themselves while accepting their presence in the landscape and influence on ecosystem service flows. Thus, while these other assets might not need to be explicitly valued in the SEEA-related capital accounts, they likely need to be considered in the assessment of ecosystem condition and capacity. This may provide an explicit link to the SNA where values and changes in values of produced assets are recorded.[33]

The multi-dimensional nature of an ecosystem raises challenges for valuing whole, spatial explicit, ecosystems as assets. On one hand, if ecosystems are defined spatially, then one might expect a downward sloping price curve for ecosystem assets reflecting substitution between different ecosystems or between ecosystem assets and other forms of capital. However, the challenge lies in the nature of substitution. Ecosystems in one location are not perfect substitutes for otherwise identical ecosystems in other locations, so we cannot expect a law of one price to hold (Addicott and Fenichel 2019; Gollier 2019). Moreover, it unlikely that one ecosystem, in the sense of common categorization, is identical to ecosystems elsewhere. Condition and components change.

Attempts to measure the change in value of multiple dimensions of ecosystems suggest a change in value (asset prices) can be measured. For example, Yun et al. (2017) measure changes in the value of the Baltic Sea focusing on three interacting fish stocks. But, Yun et al. only measure *prices* for individual fish stocks (accounting for interactions).[34] It is not clear there is a valid price concept for a multi-dimensional asset that is not actually traded as a bundle, i.e. where there are multiple characteristics contributing to a single flow of real income. The same insight comes from considering hedonic analysis (Freeman 2003; Phaneuf and Requate 2017). A hedonic price function measures the change in rental rate with respect to a change is a specific attribute, a second stage is required that links implicit price-quantity pairs to trace out the demand curve for the attribute. A key reason for the need for the second stage is "there may be substitute and complementary relationships among characteristics," (Freeman 2003).

Importantly, any single measure of price does not fully capture these effects. The reason is that price is a marginal concept. Furthermore, in order for there to be a price, we must be clear what margin is changing and being priced. In a multi-dimensional / characteristics setting, the derivative becomes a gradient or a vector of prices. In practice most hedonic work focuses on identifying the implicit price for one attribute. While hedonic functions estimate a

[33] These types of influence conceptually are captured in the parameters above, but in practice the empirical work likely only partially captures these influences.

[34] The capn package also includes the command demo("LV"), which illustrates the approximation approach for a multiple stock ecosystem, however the parameterization for this demo is only illustrative. Yun et al. hope to include an illustrate example for the multi-stock case with real data with the capn 2.0.

vector of effects, often these are included to condition variance rather than identify the marginal effect of each characteristic on the observed rental price. An additional challenge for most hedonic analysis is that they do not make use of information about the dynamics of the characteristics that priced. Finally, for most ecosystems, important changes are not happening on the extensive margin – the area of the ecosystem is not always changing. Rather, many important changes happen on the intensive margin relating to changes of the characteristics within the system itself.

An important motivation for focusing on valuing changes in ecosystem assets is that, even if ecosystem assets are made up of natural assets, the change in value of an ecosystem assets is unlikely to be the sum change in value of natural assets if the value of those natural assets are measured independently. That is **the value of an ecosystem is not the sum of its part without accounting for the interactions among those parts**. If complementarity interactions dominate then, independent measurement will underestimate the value change, and if substitution interactions dominate, then independent measurement will overestimate the value change.[35] This is why an ecosystem focused approach is imperative to avoid double counting and undercounting. Furthermore, which real income flows are counted for a given asset that produces many services matters.

In fact, this is no different than for capital allocated in the market. The key difference is that we are comfortable assuming that market allocated capital is generally efficient and the complementarity and substitution interactions are subsumed into general equilibrium price effects so we don't have worry about details of the interactions. In Yun et al.'s (2017) case, if they valued predators and prey separately, then predators would have had a greater asset price and prey a lower asset price, but the interactions between predators and prey impact the prices of these individual assets.

There are some important features of the way we might expect natural assets to behave in the context of an ecosystem. First, some stocks of natural assets will almost certainly have negative contributions in a broader ecosystem context. For example, a common predator of an endangered species may have a negative asset price because increasing the number of predators lowers the net present value to society received from the system – a very strong substitution effect. Second, much of the action may work through the complex set of capital gains components previously mentioned. Specifically, any cross effect is only present in a systems set-up.

These features have implications for how we interpret market prices for natural assets when markets do exist, e.g., land prices. SEEA-EEA has largely focused on ecosystems on land. In many countries, much land is privately owned and that land often has forests, wetlands, grasslands, and other ecosystems "on" it. So how should we think about land prices? Recall that the value of an asset is the present value of income flows. Whether or not land prices capture ecosystem services depends on whether the right to the present value of net income is actually exchanged in the market or whether market exchange of the asset influences the net present value of service flows for all claimants. For example, if I save on air conditioning because of shade from a neighbour's tree, this is an income flow that is invariant to whether my neighbour sells her property and is unlikely to influence the sale price.

Also, some services are capitalized, while others are not. For example, agriculture land prices may capture the present value of soil fertility associated with soil carbon, but are unlikely to capture the value that soils can play in sequestering carbon and mitigating climate change. The reason is that farmers only capture soil's contribution to increased yields of marketable crops, but farmers are seldom compensated for climate related benefits associated with sequestering carbon in the soil.

Some services that do not capitalize directly might capitalize via regulations, norms, or payment or contracting programs (Kinzig et al. 2011). However, in this case a market price may not reflect the asset's marginal value because behaviors – the actual tradeoffs being made – are not only driven by market incentives. In some cases, the creation of markets, e.g., tradable permits in fisheries (Grafton, Squires, and Fox 2000), can reflect the marginal value (price) of saving an addition unit of natural capital, but such markets do not exist for ecosystems per se.

[35] This may be an issue with the SEEA –CF.

Payments for ecosystem services (PES) (Ferraro and Simpson 2002; Jack, Kousky, and Sims 2008; Wunder 2005) programs do exist, but there a numerous reasons that these payments may not actually reflect the marginal value of a given ecosystem (Adamowicz et al. 2019; Salzman et al. 2018; Groom and Palmer 2010). Nevertheless, PES based prices may reflect marginal rental rates associated with land-use changes under prevailing market distortions, and these may be reasonable to use for national accounting purposes. The challenge, however, is that this are almost always payments for actions or inputs rather than true rental rates (Salzman et al. 2018), and hence the applicability will depend on the specific PES design (and payments must actually be accepted not simply offered).

Broadly, for many ecosystems the benefits are enjoyed outside of the ecosystem itself. Many institutions thus exist to protect implicit rights to those benefits, even if the beneficiaries do not own the underlying capital. Market prices will not capture this portion of expected future income flows.

As noted, SEEA-EEA has largely focused on terrestrial ecosystems, but aquatic ecosystems are also important and have unique challenges. Defining ecosystems as spatial units creates greater challenges. First, property rights regimes, and therefore allocation mechanisms, are fundamentally different in aquatic environments relative to terrestrial ones. Marine spatial planning is just now developing, there is little talk of fee simple ownerships, and understanding the economic implications of marine spatial plans will be important for their development (White, Halpern, and Kappel 2012). Indeed, water bodies often define the boundaries of spatially defined land-based property. Furthermore, the bundle of rights associate with "owning" water is general more restricted than with owning land. Second, aquatic environments are more dynamic and fluid. The set of species interactions is far more complex and the movement of currents and organisms, not to mention rivers, makes the concept of a spatial unit itself more challenging. Finally, almost all of the services related to marine ecosystems are enjoyed offsite.

Challenges associated with defining assets spatially

In practice, there are interactions to be considered in the definition of asset size, location, and value, whether a natural asset or an ecosystem asset. Location is important because most natural and ecosystem assets provide highly localized services that can be hard to arbitrage (Addicott and Fenichel 2019) and there is seldom a law of one price. However, local scale measurement can be difficult because prices often have to be imputed with statistical analysis. Assessing the asset price or change in value of an asset locally might be ideal, but as analysis is increasingly local, the number of measurements and statistical power often falls.

For ecosystems there is an additional challenge, which is that not all assets within an ecosystem operate at the same spatial scale or interact with produced or human capital to create services at the same spatial scale. Therefore, defining a unit for an ecosystem may entail defining which component assets are most important. Furthermore, the well-known modifiable area unit problem in geography and spatial analysis suggests that measurement may be highly sensitive to the scale that analysis is conducted (Jelinski and Wu 1996). Further discussion is needed to understand how SEEA-EEA capital accounts will overcome this challenge in a non-arbitrary fashion. Another challenge is that there may be ecosystems where the most material component asset is migratory. This likely an especially large challenge for marine ecosystems. The challenges of aggregation and scaling have also been identified in other ecosystem accounting research areas and will be discussed further as a distinct topic in the revision process.

5. Conclusion

Feasible methodology exists to value natural and ecosystem assets. In broad terms, this methodology aligns with economic capital theory and national accounting measurement approaches. The divergence between environmental economists and national accounts appears primarily to concern the measurement boundaries – i.e. what contributes to net present value. National accounts tend to think of this in terms of asset and production boundaries. Economists tend to think of this as concerning the definition of the social welfare or political preference function. While there is considerable commonality in the context of produced assets, in the context of natural and ecosystem assets, the former tend to be narrower for practical and historical reasons and the latter tend to be more inclusive. If we were only concerned with individual natural assets, then a consistent set of accounts could, in principle support different aggregate measures associated with different boundaries. However, this is more challenging for ecosystem assets because there are general equilibrium price effects that happen outside of the market through ecological interactions. This means that prices themselves are fundamentally connected to the boundary decisions.

Such boundary decisions are fraught but, in fact are a common operating space for official statisticians. For example, the increasing desire to focus on social progress and move "beyond GDP" (Stiglitz, Sen, and Fitoussi 2010) concerns, from an accounting perspective, shifting the asset, income, and production boundaries. Historical consistency calls for keeping the old boundaries, but technological change makes this impossible. Social change also makes keeping the old boundaries unadvisable. From an economic theory perspective, **accounting boundaries are where preference information enters the accounts.** Or, as Hulten (2006) puts it, "when it comes to capital, however, it more a question of what to do than how to do it." For example, excluding changes in the existence value of scenic places simply means giving a lower weighting to the wellbeing of individuals with high preferences for these features. Recognising this wider context of the role of accounting boundaries should be an important input to the discussion in the SEEA EEA revision. The fundamental questions is whether the accounts are meant to solely inform government "means", e.g., taxable product, or performance towards policy "ends." While, in the first half of the 20th century the accounts may have been designed to focus on means, in the early part of the 21st century there has been a clear shift towards a desire for the accounts to inform preference towards "ends."

The key messages from this paper are that welfare based measures of change can provide input prices for observed quantities of environmental goods and natural and ecosystem assets. These can be combined with index number theory to derive appropriate nominal prices for national accounts, if they are measured at broad enough scales. There are greater measurement challenges in the transferability of information from highly localized case studies, the summing public goods, and determining the extent of the market (Smith 1993). But none of these are insurmountable challenges. A slightly more difficult challenge is to determine whether changes in assets involve exogenous shifts or if the entire system is one giant endogenous general equilibrium system. However, of first order importance in developing capital accounts and balance sheets for natural and ecosystem assets is determining the boundary of what – or in fact who – counts.

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